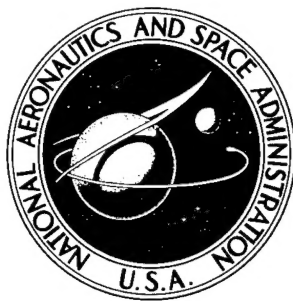
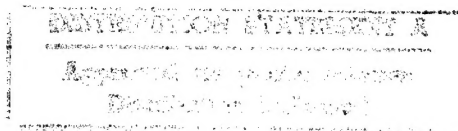


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# SUPERSONIC FLUTTER OF SIMPLY SUPPORTED ISOTROPIC SANDWICH PANELS

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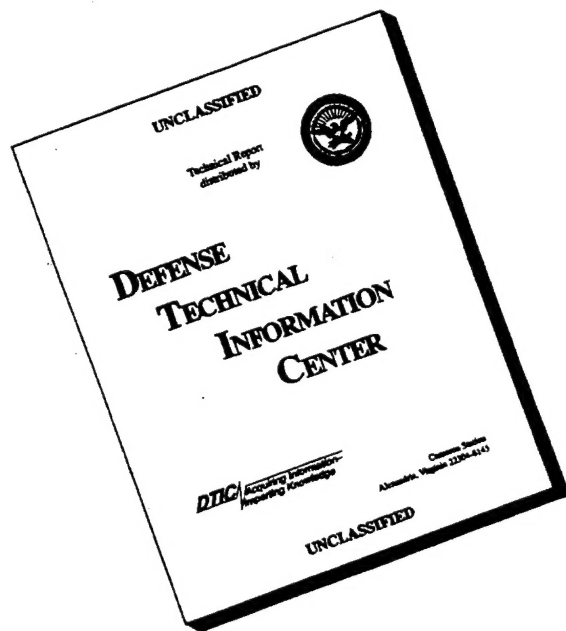
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# SUPERSONIC FLUTTER OF SIMPLY SUPPORTED ISOTROPIC SANDWICH PANELS\*

By Larry L. Erickson and Melvin S. Anderson  
Langley Research Center

## SUMMARY

[A theoretical solution using two-dimensional static aerodynamics is presented for the supersonic flutter characteristics of flat rectangular isotropic sandwich panels with simply supported edges. Tables and charts giving the values of the dynamic-pressure parameter required for flutter are presented for various values of panel length-width ratio, shear flexibility, and midplane stress. It is found that a decrease in transverse shear stiffness will usually lower the dynamic pressure required to induce flutter. However, for certain cases of midplane tension the opposite effect occurs. Panel mode shapes are also presented and a comparison is made of the two-mode Galerkin, the pre-flutter, and the exact flutter solutions.]

## INTRODUCTION

Panel flutter is an important design consideration for vehicles traveling at high Mach numbers. Consequently, considerable literature has been published dealing with several aspects of the problem. (See ref. 1.) One aspect which has not been adequately evaluated is the effect of transverse shear flexibility on the flutter characteristics of sandwich panels. Light-weight structural configurations for supersonic and hypersonic vehicles may incorporate stiffened panels such as honeycomb sandwich panels which in most cases cannot be considered rigid in shear. Therefore, unconservative designs may result if the transverse shear stiffness of such panels is not taken into account.

An estimate of the influence of shear flexibility on panel flutter was provided in reference 2 for flat and curved isotropic panels and in reference 3 for orthotropic panels where results of two-mode Galerkin solutions were presented. However, it is known that these approximate solutions become increasingly in error as the length-width ratio increases. As in references 2 and 3, it is assumed herein that the aerodynamic loading is given by two-dimensional static aerodynamics which are incorporated with the

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\*The basic theoretical development presented herein was given in a thesis by Melvin S. Anderson in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Engineering Mechanics, Virginia Polytechnic Institute, Blacksburg, Virginia, June 1965.

small-deflection theory for flat sandwich panels developed in reference 4. However, in the present investigation, the resulting differential equations are solved exactly.

The results of reference 5 indicate that for the range of parameters shown therein, panel flutter analyses based on this simple aerodynamic theory are reasonably accurate for isotropic panels without shear flexibility when compared with the results obtained by using three-dimensional unsteady aerodynamics. Thus, for most cases, the results obtained herein are not expected to differ greatly from results that would be obtained by using more accurate aerodynamic theories.

The numerical results of the analysis are presented in the form of tables and charts and are discussed. Details of the analysis appear in the appendixes.

### SYMBOLS

A,B,C coefficients appearing in equations (A6)

$$\bar{A} = \frac{\eta^4}{(1 - rk_X)} \left[ k_X - 2n^2 + r(n^2 k_X + \phi) \right]$$

a length of panel

$$\bar{B} = \frac{\eta^4}{1 - rk_X} \left[ \phi(1 + n^2 r) - n^4 \right]$$

b width of panel

D flexural stiffness of isotropic sandwich panel,

$$\frac{E_f t_f h_c^2 \left( 1 + \frac{t_f}{h_c} \right)^2}{2(1 - \mu^2)} + \frac{E_f t_f^3}{6(1 - \mu^2)}$$

$D_Q$  transverse shear stiffness of isotropic sandwich panel,  $G_c h_c \left( 1 + \frac{t_f}{h_c} \right)^2$

$D_1, D_2, D_3$  coefficients defined by equations (A15)

$E_f$  Young's modulus for faces of isotropic sandwich panel

$F( )$  function defined by equation (A25)

$G_c$  shear modulus for core of isotropic sandwich panel

|                                      |   |
|--------------------------------------|---|
| $h_c$                                | depth of sandwich core  |
| $j, m, n$                            | integers  |
| $k_x, k'_x$                          | midplane stress parameters, $\frac{N_x b^2}{\pi^2 D}$ , $\frac{N_x a^2}{\pi^2 D}$   |
| $l$                                  | lateral aerodynamic loading   |
| $M$                                  | free-stream Mach number   |
| $M_x, M_y$                           | intensity of internal bending moments acting upon a cross section originally parallel to the $yz$ and $xz$ planes, respectively   |
| $M_{xy}$                             | intensity of internal twisting moment acting in a cross section originally parallel to $yz$ plane or $xz$ plane                   |
| $\bar{m}$                            | exponent in equations (A6), denotes roots of equation (A9) when used with subscript $j$   |
| $N_x, N_y$                           | intensity of middle plane forces parallel to $x$ and $y$ axes, respectively (positive in compression)                             |
| $P = \frac{r}{1+r}$                  |   |
| $Q_x, Q_y$                           | intensity of internal shears acting in $z$ -direction in cross sections originally parallel to $yz$ and $xz$ planes, respectively |
| $q$                                  | free-stream dynamic pressure, $\frac{1}{2} \rho_a V^2$  |
| $R = \frac{r/\eta^2}{1+r}$           |   |
| $r, r'$                              | shear flexibility parameters, $\frac{\pi^2 D}{b^2 D_Q}$ , $\frac{\pi^2 D}{a^2 D_Q}$   |
| $S, T$                               | coefficients defined by equations (A30)   |
| $s = P \cdot \frac{\bar{A}}{\eta^2}$ |   |
| $t$                                  | time  |

$t_f$  thickness of sandwich face plates

$V$  free-stream velocity of airflow

$w$  deflection of middle surface of plate, measured in z-direction

$x, y, z$  orthogonal coordinates (see fig. 1)

$\alpha, \delta, \epsilon$  assumed components of roots of equation (A9) (See eqs. (A11))

$$\beta = \sqrt{M^2 - 1}$$

$$\bar{\beta}_j = \frac{\bar{m}_j}{\pi \eta}$$

$\Gamma$  coefficient defined by equations (B6)

$$\gamma = \frac{\lambda \eta r}{4\pi^2 (1 - rk_x)}$$

$$\xi = \gamma - \alpha$$

$\eta$  panel length-width ratio,  $\frac{a}{b}$

$\lambda, \lambda'$  dynamic pressure parameters,  $\frac{2qb^3}{\beta D}$ ,  $\frac{2qa^3}{\beta D}$

$$\bar{\lambda} = \lambda \eta^3 \frac{1 + n^2 r}{1 - rk_x}$$

$\mu$  Poisson's ratio for sandwich panel, defined in terms of curvatures

$$\xi = \gamma + \alpha$$

$\rho_a$  free-stream mass density of air

$\rho_m$  mass density per unit area of panel

$\phi, \phi'$  frequency parameters,  $\left(\frac{\omega}{\omega_r}\right)^2 + n^2 \frac{N_y b^2}{\pi^2 D}$  and  $\left(\frac{\omega}{\omega_r}\right)^2 + n^2 \frac{N_y a^2}{\pi^2 D}$

$$\frac{1}{\psi_j} = \left( \frac{1}{1 - \beta_j^2} \right) + r$$

$$\Omega = \tan^{-1} R$$

$\omega$  panel frequency

$\omega_r, \omega'_r$  fundamental frequency of simply supported panel rigid in shear with an infinite length or width, respectively,  $\sqrt{\frac{\pi^4 D}{b^4 \rho_m}}, \sqrt{\frac{\pi^4 D}{a^4 \rho_m}}$

$\left[ \right]$  square matrix

$\left\{ \right\}$  column matrix

Subscripts:

$\infty$  evaluated at  $\frac{a}{b} = \infty$

cr denotes flutter value of parameters

p denotes preflutter value

A comma followed by a subscript denotes differentiation with respect to the subscript.

## THEORY AND ASSUMPTIONS

The configuration analyzed is shown in figure 1. It consists of a flat rectangular sandwich panel mounted on simple supports. The panel core and face materials are isotropic; hence, the panel itself is referred to as being isotropic. The panel has a length  $a$  and a width  $b$  and is subjected to uniform midplane force intensities  $N_x$  and  $N_y$  (positive in compression). The supersonic flow at Mach number  $M$  is over the top surface of the panel and is parallel to the  $x$ -axis.

The analysis of the configuration is based on the small deflection theory for flat sandwich panels developed in reference 4. This theory incorporates the effect of shear deformations by expressing the total panel curvature in the  $x$ - or  $y$ -direction and the twisting distortion as the sum of the contributions made by each of the internal



shears ( $Q_x, Q_y$ ) and moments ( $M_x, M_y, M_{xy}$ ). The resulting force-distortion equations can be solved for the three separate moments. Substitution of these expressions for the moments into the equations for equilibrium of moments about the x- and y-axes and the equation for equilibrium of vertical forces yields three independent equations relating the lateral displacement  $w$  and the two average shear angles  $Q_x/D_Q$  and  $Q_y/D_Q$ . These equations include the in-plane force intensities and the lateral loading. In the analysis presented herein, the lateral loading is comprised of the inertia force and the pressure due to supersonic flow (given by two-dimensional static aerodynamics). In-plane and rotary inertia loadings are not considered.

The boundary conditions imposed are those of simple supports, but inclusion of shear effects requires that a third boundary condition be satisfied in addition to the two usual conditions of zero moment and middle-surface deflection along the panel edges. This third boundary condition depends on the assumption made for the panel support. If the support is applied only to the middle surface of the panel, the boundary condition is that  $M_{xy}$  must vanish. If the support is assumed to be applied over the entire thickness of the panel the shear angle  $Q_x/D_Q$  is zero along an edge parallel to the x-axis because there is no x-displacement of points at the boundary. Similarly, the shear angle  $Q_y/D_Q$  is zero along an edge parallel to the y-axis. The last boundary condition (shear angle of  $0^\circ$ ) is usually the more closely approached in practice and is the one used herein. The exact solutions to the differential equations, subject to the stated boundary conditions, lead to a transcendental characteristic equation from which the panel frequencies and mode shapes can be determined as a function of midplane loads and dynamic pressure.

For the type of aerodynamic-force approximation used herein, flutter is known to occur only if variations of airflow or panel parameters can force a coalescence of two panel frequencies (ref. 6). The locus of such points forms a flutter boundary that separates a region of stable motion, where all frequencies are real and distinct, from a region of dynamic instability where at least one pair of frequencies are complex conjugates. Details of the exact solution together with a practical procedure for obtaining numerical flutter results are presented in appendix A.

## RESULTS AND DISCUSSION

For a panel of given length or width, the flutter value of the dynamic pressure  $q$  is a function of the length-width ratio  $a/b$ , the bending and shear stiffnesses  $D$  and  $D_Q$ , respectively, and the in-plane load  $N_x$ . For the presentation of results, these variables are expressed in terms of nondimensional parameters.

Numerical results of the analysis are presented in tabular form in tables I and II. Table I is for  $a/b < 1$ , where flutter values of the dynamic-pressure parameter

$\lambda'_{cr} = \frac{2qa^3}{\beta D}$  and the frequency parameter  $\phi'_{cr} = \frac{\rho_m a^4}{\pi^4 D} \omega^2 + \frac{a^2}{\pi^2 D} N_y$  are tabulated for

various values of the shear-flexibility parameter  $r' = \frac{\pi^2 D}{a^2 D_Q}$  and the stress parameter

$k'_x = \frac{N_x a^2}{\pi^2 D}$ . The flutter values  $\lambda'_{cr}$  were obtained by plotting frequency loops ( $\lambda'$  against

$\phi'$ , see fig. 2) for constant values of  $r'$ ,  $k'_x$ , and  $a/b$ . At the peak of the frequency loop  $\lambda' = \lambda'_{cr}$  and any increase in  $\lambda'$  produces flutter. The flutter values of  $\phi'$  associated with  $\lambda'_{cr}$  incorporate  $N_y$  and can be used to determine flutter frequencies  $\omega_{cr}$ . As in the case for panels which are rigid in shear, the flutter value of  $q$  is not affected by  $N_y$ . Table II is for  $\frac{a}{b} \geq 1$  and to keep the numerical values of the tabulated results within reasonable size, the parameters have been redefined in terms of  $b$ .

Thus, in table II, flutter values of  $\lambda_{cr} = \frac{2qb^3}{\beta D}$  and  $\phi_{cr} = \frac{\rho_m b^4}{\pi^4 D} \omega^2 + \frac{b^2}{\pi^2 D} N_y$  are tabulated for values of  $r = \frac{\pi^2 D}{b^2 D_Q}$  and  $k_x = \frac{N_x b^2}{\pi^2 D}$ . Note that all results are presented in terms of

the shorter dimension of the panel. Tables I and II also contain values of a parameter  $\alpha$  which can be used to calculate flutter mode shapes. (See appendix A.) In table II, results are not presented for any value of  $a/b$  greater than that required to produce  $\lambda_{cr} = 0$ .

The results in tables I and II are presented graphically in figures 3 and 4. To keep these figures within reasonable size,  $(\lambda')^{1/3}_{cr}$  is plotted against  $a/b$  when  $0 \leq a/b \leq 1$  and  $\lambda_{cr}^{1/3}$  is plotted against  $b/a$  when  $1 \leq a/b \leq \infty$  for different values of the shear-flexibility parameters  $r'$ ,  $r$ . The flutter boundaries appearing in each individual figure correspond to different values of the stress parameters  $k'_x$ ,  $k_x$ . Figure 4 is for stress-free panels with various values of  $r'$  and  $r$ . It shows that  $\lambda'_{cr}$  is essentially independent of the panel width for  $a/b \leq 1/4$  and that  $\lambda_{cr}$  is essentially independent of length for  $a/b \geq 10$ . Figure 4 also illustrates that a significant reduction in  $\lambda_{cr}$  can be caused by shear flexibility when the panel carries no in-plane loads in the  $x$ -direction. It should be noted that for  $r = 0$  ( $D_Q = \infty$ ), the panel is rigid in shear and for this case the results agree with those of reference 6.

#### Effect of Shear Stiffness and Stress on Flutter Boundaries

Figure 5 illustrates the effect of shear stiffness on  $\lambda_{cr}$  for a square panel. For a compressive force in the flow direction ( $k_x > 0$ ), the theory predicts that as the panel is made less stiff in shear ( $r$  increasing),  $\lambda_{cr}$  will decrease. The same result holds for the stress-free panel. However, when the panel is in tension ( $k_x < 0$ ), there are cases when theory predicts that a panel which is flexible in shear ( $r > 0$ ) will flutter at a higher value of  $\lambda_{cr}$  than a panel of the same geometry which is rigid in shear ( $r = 0$ ).

For example, when  $k_x = -4$ , it is seen that after a slight initial decrease,  $\lambda_{cr}$  increases steadily as the panel becomes more flexible in shear. At  $r = 2$ ,  $\lambda_{cr} = 1230$  which is over one-third larger than the  $r = 0$  value of  $\lambda_{cr}$ . That an increase in panel stiffness can lower  $\lambda_{cr}$  is an unexpected theoretical result. Whether such a physical phenomena does actually occur or whether the theory employed does not accurately represent the panel behavior under certain combinations of shear stiffness and tensile loading is not known. Consequently, such results should be regarded cautiously until refinements in the theory (for example, consideration of rotary inertia) or experimental evidence either confirm or refute this anomalous behavior.

The analysis presented in appendix A is based on the criteria that flutter occurs when two panel frequencies coalesce. Any circumstances that cause two in-vacuo frequencies to coincide will then produce, by this definition, flutter even though the dynamic pressure approaches zero. Inclusion of either aerodynamic or structural damping in theoretical flutter analyses of panels which are rigid in shear has been shown to remove these zero-dynamic-pressure flutter points (ref. 5); the resulting theoretical flutter boundaries, however, still do not compare well with experimental boundaries. Thus, to gain an idea of conditions for which the theoretical results can no longer be considered reliable, it is important to know the combinations of  $r$ ,  $k_x$ , and  $a/b$  which produce zero-dynamic-pressure flutter points. These combinations can be determined from the in-vacuo vibration characteristics of the panel as described in appendix C. The primary results are shown in figure 6. When  $k_x = \frac{2+r}{(1+r)^2}$ ,  $\lambda_{cr}$  approaches zero as  $a/b$  approaches infinity. If  $k_x$  is greater than  $\frac{2+r}{(1+r)^2}$ , then  $\lambda_{cr}$  goes to zero at values of  $a/b$  given by equation (C5) in appendix C. If  $k_x$  is less than  $\frac{2+r}{(1+r)^2}$ , then  $\lambda_{cr}$  never reaches zero but has a finite asymptotic value as  $a/b$  approaches infinity.

#### Comparison of Exact Results With Approximate Solutions

Modal solution.- Development of the exact solution provides an opportunity for evaluating the approximate two-mode Galerkin solution of reference 2. A comparison of flutter boundaries is shown in figure 7 for stress-free panels. Note that the two-mode solution becomes less accurate as  $r$  increases. Also, as in the  $r = 0$  case, it becomes less accurate as  $a/b$  increases. At  $a/b = \infty$ , the two-mode solution predicts that  $\lambda_{cr} = 0$ .

An explanation for the growing inaccuracy of the two-mode solution with increasing shear flexibility is indicated by the exact flutter mode shapes for square stress-free panels shown in figure 8. (These shapes were calculated (from eq. (A29)) by using the values of  $\alpha$  given in tables I and II.) As the panel becomes weaker in shear, the point of

maximum amplitude moves toward the trailing edge of the panel. Thus, it would be expected that the representation of the true mode shape by only two terms in a sine series, as was done in reference 2, would lead to increasingly inaccurate results as  $r$  increases.

Preflutter solution.- In reference 7, a solution to the differential equation of motion for simply supported panels that are rigid in shear was made and it yielded simple algebraic expressions for  $\lambda$  and  $\phi$ . This solution corresponds to a point on the frequency loop which is not necessarily at the peak; thus, a value of  $\lambda$  is given which is less than or equal to  $\lambda_{cr}$ , hence the name "preflutter." When transverse shear effects are considered, it is found that a simple preflutter solution still exists (appendix B), even though two additional differential equations and an additional boundary condition must be satisfied.

As can be seen from figure 9, which is for stress-free panels, the preflutter solution gives values of  $\lambda$  that are very close to  $\lambda_{cr}$  when  $a/b$  is sufficiently large. For small values of  $a/b$  the preflutter solution is often in poor agreement with the exact solution. Its behavior is typical of that shown in figure 10 which is for  $r = 2.0$ . When  $a/b = 1$ , the preflutter solution corresponds to a point low on the second leg of the frequency loop. As  $a/b$  increases, this point moves along the second leg until at  $a/b = 4$  it is nearly at the top of the frequency loop and is thus virtually identical with  $\lambda_{cr}$ . As  $a/b$  continues to increase, the preflutter point moves to the top of the loop (becoming exactly equal to  $\lambda_{cr}$ ) and then starts down the first leg. Once on the first leg, however, it stays relatively close to the top of the loop, and thus gives a close approximation to  $\lambda_{cr}$ .

For increasingly large values of  $a/b$ , it becomes more difficult to obtain numerical results from the exact solution for  $\lambda_{cr}$ . Hence, it is desirable to be able to use the simpler preflutter solution in the range where it agrees closely with the exact solution. Table III shows a comparison of the preflutter solution with  $\lambda_{cr}$  at  $a/b = 20$  for the values of  $r$  and  $k_x$  which are presented in table II. In all cases the solutions differ by less than 2 percent. Thus, the preflutter results used in table II and in figures 2 and 3 for  $a/b > 20$  are justified as being good approximations to  $\lambda_{cr}$ .

### CONCLUDING REMARKS

Charts have been presented to facilitate the determination of theoretical values of the dynamic pressure required to produce flutter of flat rectangular isotropic sandwich panels with simply supported edges. The flutter value of the dynamic pressure for a panel of given length or width is found to be a function of the length-width ratio, the bending and shear stiffnesses, and the in-plane load acting parallel to the airflow. It is independent of the in-plane load acting perpendicular to the airflow as long as buckling does not occur.

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The theory predicts that a reduction in transverse shear stiffness usually causes a panel to be more susceptible to flutter. An unusual result is obtained for panels which carry tension loads in the direction parallel to the airflow. In these cases it is theoretically possible for a reduction in shear stiffness to make a panel less susceptible to flutter. However, this result should be regarded cautiously since at present there is no experimental evidence available with which to compare such theoretical behavior.

A comparison is made between the exact solution for the flutter dynamic pressure and two approximate solutions. A two-mode Galerkin solution is shown to become increasingly inaccurate as a panel becomes more flexible in shear. A relatively simple preflutter expression is found to give results that are in good agreement with the exact solution for sufficiently large length-width ratios.

Langley Research Center,  
National Aeronautics and Space Administration,  
Langley Station, Hampton, Va., November 8, 1965.

## APPENDIX A

### EXACT SOLUTION FOR THE SUPERSONIC FLUTTER BEHAVIOR OF SANDWICH PANELS

#### Differential Equations

The small-deflection equilibrium equations used herein to represent the behavior of a sandwich panel are derived in reference 4 by use of the following force-distortion equations which relate moments  $M_x, M_y, M_{xy}$ , shears  $Q_x, Q_y$ , and displacement  $w$ :

$$\left. \begin{aligned} M_x &= -D \left[ \frac{\partial}{\partial x} \left( \frac{\partial w}{\partial x} - \frac{Q_x}{D_Q} \right) + \mu \frac{\partial}{\partial y} \left( \frac{\partial w}{\partial y} - \frac{Q_y}{D_Q} \right) \right] \\ M_y &= -D \left[ \frac{\partial}{\partial y} \left( \frac{\partial w}{\partial y} - \frac{Q_y}{D_Q} \right) + \mu \frac{\partial}{\partial x} \left( \frac{\partial w}{\partial x} - \frac{Q_x}{D_Q} \right) \right] \\ M_{xy} &= \frac{1 - \mu}{2} D \left[ \frac{\partial}{\partial x} \left( \frac{\partial w}{\partial y} - \frac{Q_y}{D_Q} \right) + \frac{\partial}{\partial y} \left( \frac{\partial w}{\partial x} - \frac{Q_x}{D_Q} \right) \right] \end{aligned} \right\} \quad (A1)$$

where the bending and shear stiffnesses as given in reference 8 are

$$\left. \begin{aligned} D &= \frac{E_f t_f h_c^2 \left( 1 + \frac{t_f}{h_c} \right)^2}{2(1 - \mu)^2} + \frac{E_f t_f^3}{6(1 - \mu)^2} \\ D_Q &= G_c h_c \left( 1 + \frac{t_f}{h_c} \right)^2 \end{aligned} \right\} \quad (A2)$$

respectively, and  $\mu$  is Poisson's ratio. The assumptions involved in equations (A1) require that a straight line perpendicular to the undeformed middle surface of the panel remain straight and of constant length after deformation but not necessarily perpendicular to the deformed middle surface. This inclination in the  $x$ - (or  $y$ -) direction from a right angle is the average shear angle  $\frac{Q_x}{D_Q}$  (or  $\frac{Q_y}{D_Q}$ ).

The lateral aerodynamic pressure given by static linearized two-dimensional supersonic flow theory is

$$l = \frac{-2q}{\beta} \frac{\partial w}{\partial x} \quad (A3)$$

where  $q = \frac{1}{2} \rho_a V^2$  is the free-stream value of the dynamic pressure and  $\beta = \sqrt{M^2 - 1}$ .

## APPENDIX A

Substituting the above expression for  $l$ , together with the lateral inertia loading, into the three equilibrium equations of reference 4 yields

$$\left. \begin{aligned} -N_x w_{,xx} - N_y w_{,yy} + Q_{x,x} + Q_{y,y} - \rho_m w_{,tt} - \frac{2q}{\beta} w_{,x} &= 0 \\ -w_{,xyy} - w_{,xxx} - \frac{Q_x}{D} + \frac{1}{DQ} \left[ Q_{x,xx} + \left( \frac{1-\mu}{2} \right) Q_{x,yy} + \left( \frac{1+\mu}{2} \right) Q_{y,xy} \right] &= 0 \\ -w_{,xxy} - w_{,yyy} - \frac{Q_y}{D} + \frac{1}{DQ} \left[ Q_{y,yy} + \left( \frac{1-\mu}{2} \right) Q_{y,xx} + \left( \frac{1+\mu}{2} \right) Q_{x,xy} \right] &= 0 \end{aligned} \right\} \quad (A4)$$

where  $\rho_m$  is the mass per unit area of the panel and  $N_x$  and  $N_y$  are positive in compression.

### Satisfaction of Boundary Conditions Along the Streamwise Edges

For the simply supported edges parallel to the x-axis at which the support is applied over the entire thickness, the boundary conditions are (see ref. 4)

$$\left. \begin{aligned} w &= 0 \\ M_y &= 0 \\ \frac{Q_x}{DQ} &= 0 \end{aligned} \right\} \quad (A5)$$

General product solutions for the lateral deflection and the shears which satisfy these boundary conditions are

$$\left. \begin{aligned} w(x,y,t) &= A e^{\frac{\bar{m}x}{a}} \sin n\pi \frac{y}{b} e^{i\omega t} \\ Q_x(x,y,t) &= B e^{\frac{\bar{m}x}{a}} \sin n\pi \frac{y}{b} e^{i\omega t} \\ Q_y(x,y,t) &= C e^{\frac{\bar{m}x}{a}} \cos n\pi \frac{y}{b} e^{i\omega t} \end{aligned} \right\} \quad (A6)$$

where  $n$  is an integer indicating the number of sinusoidal half-waves in the y-direction and  $\omega$  is the panel frequency. The expressions for  $w$ ,  $Q_x$ , and  $Q_y$  also satisfy the differential equations (A4) provided that

# APPENDIX A

$$\begin{bmatrix} \left(-k_x \frac{\bar{m}^2}{\pi^2} + \phi \eta^2 - \frac{\lambda \eta \bar{m}}{\pi^4}\right) & \frac{\bar{m}}{\pi} & -n \\ \frac{\bar{m}}{\pi} \left(n^2 \eta^2 - \frac{\bar{m}^2}{\pi^2}\right) & -\eta^2 + r \frac{\bar{m}^2}{\pi^2} - \left(\frac{1-\mu}{2}\right) \eta^2 n^2 r & -\frac{\bar{m}}{\pi} \left(\frac{1+\mu}{2}\right) nr \\ n \left(n^2 \eta^2 - \frac{\bar{m}^2}{\pi^2}\right) & \frac{\bar{m}}{\pi} \left(\frac{1+\mu}{2}\right) nr & -(1+n^2 r) + \left(\frac{1-\mu}{2}\right) \frac{r}{\eta^2} \frac{\bar{m}^2}{\pi^2} \end{bmatrix} \begin{Bmatrix} A \\ \frac{ab^2}{\pi^3 D} B \\ \frac{a^2 b}{\pi^3 D} C \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad (A7)$$

where

$$\left. \begin{aligned} \eta &= \frac{a}{b} \\ k_x &= \frac{N_x b^2}{\pi^2 D} \\ r &= \frac{\pi^2 D}{b^2 D_Q} \\ \lambda &= \frac{2qb^3}{\beta D} \\ \phi &= \left(\frac{\omega}{\omega_r}\right)^2 + n^2 \frac{N_y b^2}{\pi^2 D} \\ \omega_r^2 &= \frac{\pi^4 D}{b^4 \rho_m} \end{aligned} \right\} \quad (A8)$$

Nontrivial solutions are obtained by equating the determinant of the matrix in equation (A7) to zero. Expanding this determinant leads to

$$\left(\bar{m}^4 - 4\gamma \bar{m}^3 + \pi^2 \bar{A} \bar{m}^2 + \bar{\lambda} \bar{m} - \pi^4 \bar{B}\right) \left[1 - \frac{n^2 r}{\eta^2} \left(\frac{1-\mu}{2}\right) \left(\frac{\bar{m}^2}{\pi^2 n^2} - \eta^2\right)\right] = 0 \quad (A9)$$



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where

$$\left. \begin{aligned} \gamma &= \frac{\lambda \eta r}{4\pi^2(1 - rk_X)} \\ \bar{A} &= \frac{\eta^2}{1 - rk_X} \left[ k_X - 2n^2 + r(n^2 k_X + \phi) \right] \\ \bar{\lambda} &= \lambda \eta^3 \frac{1 + n^2 r}{1 - rk_X} \\ \bar{B} &= \frac{\eta^4}{1 - rk_X} \left[ \phi(1 + n^2 r) - n^4 \right] \end{aligned} \right\} \quad (A10)$$

If the roots of equation (A9) are written in a form similar to that used in reference 6,

$$\left. \begin{aligned} \bar{m}_1 &= \gamma + \alpha + i\delta \\ \bar{m}_2 &= \gamma + \alpha - i\delta \\ \bar{m}_3 &= \gamma - \alpha + \epsilon \\ \bar{m}_4 &= \gamma - \alpha - \epsilon \\ \bar{m}_5 &= \pi \eta n \left[ 1 + \frac{2}{(1 - \mu)n^2 r} \right]^{1/2} \\ \bar{m}_6 &= -\bar{m}_5 \end{aligned} \right\} \quad (A11)$$

they can be related to the coefficients in equation (A9) as follows:

$$6\gamma^2 - 2\alpha^2 + \delta^2 - \epsilon^2 = \pi^2 \bar{A} \quad (A12a)$$

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$$2\alpha(\delta^2 + \epsilon^2) - 2\gamma\left[\delta^2 - \epsilon^2 + 2(\gamma^2 - \alpha^2)\right] = \bar{\lambda} \quad (\text{A12b})$$

$$\gamma^4 + \gamma^2(\delta^2 - \epsilon^2 - 2\alpha^2) - 2\alpha\gamma(\epsilon^2 + \delta^2) + (\alpha^2 - \epsilon^2)(\alpha^2 + \delta^2) = -\pi^4\bar{B} \quad (\text{A12c})$$

For computational purposes it is convenient to solve equations (A12a) and (A12b) for  $\delta$  and  $\epsilon$ .

$$\left. \begin{aligned} \delta^2 &= \frac{\bar{\lambda}}{4\alpha} + \alpha^2 + \frac{\pi^2\bar{A}}{2} - \frac{\gamma}{\alpha}\left(2\gamma^2 + 3\gamma\alpha - \pi^2\frac{\bar{A}}{2}\right) \\ \epsilon^2 &= \frac{\bar{\lambda}}{4\alpha} - \alpha^2 - \frac{\pi^2\bar{A}}{2} - \frac{\gamma}{\alpha}\left(2\gamma^2 - 3\gamma\alpha - \pi^2\frac{\bar{A}}{2}\right) \end{aligned} \right\} \quad (\text{A13})$$

It should be noted that  $\delta^2$  and  $\epsilon^2$  could be negative and this possibility was allowed for in the calculations by taking into account any imaginary quantities that appear. However, for the ranges of parameters covered in the calculations,  $\delta$  was real for any point on the frequency loop and  $\epsilon$  was real except at sufficiently high axial compression where  $\epsilon$  became imaginary. Using these relations to eliminate  $\delta$  and  $\epsilon$  from equation (A12c) yields the following cubic in  $\alpha^2$

$$\alpha^6 - D_1\alpha^4 + D_2\alpha^2 - D_3 = 0 \quad (\text{A14})$$

where

$$\left. \begin{aligned} D_1 &= 3\gamma^2 - \frac{\pi^2\bar{A}}{2} \\ D_2 &= \left(2\gamma^2 - \frac{\pi^2\bar{A}}{4}\right)^2 + \frac{1}{4}\left(\pi^4\bar{B} - \gamma\bar{\lambda}\right) - \gamma^4 \\ D_3 &= \left[\frac{\bar{\lambda}}{8} - \gamma\left(\gamma^2 - \frac{\pi^2\bar{A}}{4}\right)\right]^2 \end{aligned} \right\} \quad (\text{A15})$$

Equations (A13) and (A14) provide a procedure for obtaining numerical values of  $\lambda$  once the boundary conditions at  $x = 0$  and  $x = a$  are satisfied.

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### Satisfaction of Boundary Conditions Along the Leading and Trailing Edges

For the simply supported edges parallel to the y-axis at which the support is applied over the entire thickness, the boundary conditions are the same as those given by equations (A5) if y and x are interchanged. Before the boundary conditions can be applied to the solutions given by equations (A6), it is necessary to express w,  $Q_x$ , and  $Q_y$  in terms of one set of coefficients (that is, either A, B, or C). The coefficient A can be eliminated from the second and third rows of matrix equation (A7). The result is

$$\left[ 1 - \frac{n^2 r}{\eta^2} \left( \frac{1 - \mu}{2} \right) \left( \frac{\bar{m}_j^2}{n^2 \pi^2} - \eta^2 \right) \right] \left[ \eta B_j - \frac{\bar{m}_j}{n \pi} C_j \right] = 0 \quad (A16)$$

When j equals 5 or 6, the first bracketed term is equal to zero (eqs. (A9) and (A11)) and no information is obtained about the relationship between  $B_j$  and  $C_j$ . However, when  $j = 1, 2, 3, 4$ , the first bracketed term is not, in general, equal to zero. Therefore,

$$C_j = \frac{\pi \eta n}{\bar{m}_j} B_j \quad (j = 1, 2, 3, 4) \quad (A17)$$

Replacing  $C_j$  with  $\frac{\pi \eta n}{\bar{m}_j} B_j$  in the third row of equation (A7) gives

$$A_j = \left[ \frac{\pi^2 \eta^2 + r(\pi^2 n^2 \eta^2 - \bar{m}_j^2)}{\bar{m}_j \pi^2 (\pi^2 n^2 \eta^2 - \bar{m}_j^2)} \right] \frac{ab^2}{D} \cdot B_j \quad (j = 1, 2, 3, 4) \quad (A18)$$

If  $C_j$  is eliminated from the first and second of equations (A7), it is found that

$$A_j \left\{ -k_x \frac{\bar{m}_j^2}{\pi^2} + \phi \eta^2 - \frac{\lambda \eta \bar{m}_j}{\pi^4} \left[ \left( \frac{1 + \mu}{2} \right) \frac{\bar{m}_j r}{\pi} + \frac{\bar{m}_j}{\pi} \left( \frac{\bar{m}_j^2}{\pi^2} - n^2 \eta^2 \right) \right] \right\} = \frac{-a^3}{\pi^3 D} \left[ 1 - \frac{n^2 r}{\eta^2} \left( \frac{1 - \mu}{2} \right) \left( \frac{\bar{m}_j^2}{n^2 \pi^2} - \eta^2 \right) \right] B_j \quad (A19)$$

Again, the bracketed term before  $B_j$  is zero for j equal to 5 or 6. The coefficient of  $A_j$  is not, in general, equal to zero and therefore

$$A_5 = A_6 = 0 \quad (A20)$$

Thus, for j equal to 5 and 6, the first of equation (A7) gives

$$C_j = \frac{\bar{m}_j}{n \eta \pi} B_j \quad (A21)$$

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Use of these relations enables equations (A6) to be written in terms of  $B_j$  alone as follows:

$$\left. \begin{aligned} w &= \sum_{j=1}^4 \left[ \frac{\pi^2 \eta^2 + r(\pi^2 n^2 \eta^2 - \bar{m}_j^2)}{\bar{m}_j \pi^2 (\pi^2 n^2 \eta^2 - \bar{m}_j^2)} \right] \frac{ab^2}{D} B_j e^{\bar{m}_j \frac{x}{a}} \sin n\pi \frac{y}{b} e^{i\omega t} \\ Q_x &= \sum_{j=1}^6 B_j e^{\bar{m}_j \frac{x}{a}} \sin n\pi \frac{y}{b} e^{i\omega t} \\ Q_y &= \left[ \sum_{j=1}^4 \frac{\pi n \eta}{\bar{m}_j} B_j e^{\bar{m}_j \frac{x}{a}} + \sum_{j=5}^6 \frac{\bar{m}_j}{\pi n \eta} B_j e^{\bar{m}_j \frac{x}{a}} \right] \cos n\pi \frac{y}{b} e^{i\omega t} \end{aligned} \right\} \quad (A22)$$

Substituting the above expressions for  $w$ ,  $Q_x$ , and  $Q_y$  into the boundary condition equations yields

$$\left[ \begin{array}{cccccc} \left( \frac{1}{n^2 - \bar{\beta}_1^2} + r \right) \frac{1}{\bar{\beta}_1} & \left( \frac{1}{n^2 - \bar{\beta}_2^2} + r \right) \frac{1}{\bar{\beta}_2} & \left( \frac{1}{n^2 - \bar{\beta}_3^2} + r \right) \frac{1}{\bar{\beta}_3} & \left( \frac{1}{n^2 - \bar{\beta}_4^2} + r \right) \frac{1}{\bar{\beta}_4} & 0 & 0 \\ \left( \frac{1}{n^2 - \bar{\beta}_1^2} + r \right) \frac{e^{\bar{m}_1}}{\bar{\beta}_1} & \left( \frac{1}{n^2 - \bar{\beta}_2^2} + r \right) \frac{e^{\bar{m}_2}}{\bar{\beta}_2} & \left( \frac{1}{n^2 - \bar{\beta}_3^2} + r \right) \frac{e^{\bar{m}_3}}{\bar{\beta}_3} & \left( \frac{1}{n^2 - \bar{\beta}_4^2} + r \right) \frac{e^{\bar{m}_4}}{\bar{\beta}_4} & 0 & 0 \\ \frac{-\bar{\beta}_1}{n^2 - \bar{\beta}_1^2} & \frac{-\bar{\beta}_2}{n^2 - \bar{\beta}_2^2} & \frac{-\bar{\beta}_3}{n^2 - \bar{\beta}_3^2} & \frac{-\bar{\beta}_4}{n^2 - \bar{\beta}_4^2} & r\bar{\beta}_5 & r\bar{\beta}_6 \\ \frac{-\bar{\beta}_1 e^{\bar{m}_1}}{n^2 - \bar{\beta}_1^2} & \frac{-\bar{\beta}_2 e^{\bar{m}_2}}{n^2 - \bar{\beta}_2^2} & \frac{-\bar{\beta}_3 e^{\bar{m}_3}}{n^2 - \bar{\beta}_3^2} & \frac{-\bar{\beta}_4 e^{\bar{m}_4}}{n^2 - \bar{\beta}_4^2} & r\bar{\beta}_5 e^{\bar{m}_5} & r\bar{\beta}_6 e^{\bar{m}_6} \\ \frac{n}{\bar{\beta}_1} & \frac{n}{\bar{\beta}_2} & \frac{n}{\bar{\beta}_3} & \frac{n}{\bar{\beta}_4} & \frac{\bar{\beta}_5}{n} & \frac{\bar{\beta}_6}{n} \\ \frac{n e^{\bar{m}_1}}{\bar{\beta}_1} & \frac{n e^{\bar{m}_2}}{\bar{\beta}_2} & \frac{n e^{\bar{m}_3}}{\bar{\beta}_3} & \frac{n e^{\bar{m}_4}}{\bar{\beta}_4} & \frac{\bar{\beta}_5 e^{\bar{m}_5}}{n} & \frac{\bar{\beta}_6 e^{\bar{m}_6}}{n} \end{array} \right] \begin{Bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \\ B_5 \\ B_6 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (A23)$$

where  $\bar{\beta}_j = \frac{\bar{m}_j}{\pi \eta}$ .

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The nontrivial solution for the deflected shape of the panel is obtained by equating the determinant of this matrix to zero. By manipulating rows and columns, additional zeroes can be introduced into columns 5 and 6 of this determinant so that it becomes

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ e^{\bar{m}_1} & e^{\bar{m}_2} & e^{\bar{m}_3} & e^{\bar{m}_4} \\ \bar{m}_1^2 & \bar{m}_2^2 & \bar{m}_3^2 & \bar{m}_4^2 \\ \bar{m}_1^2 e^{\bar{m}_1} & \bar{m}_2^2 e^{\bar{m}_2} & \bar{m}_3^2 e^{\bar{m}_3} & \bar{m}_4^2 e^{\bar{m}_4} \end{vmatrix} = 0 \quad (A24)$$

Note that equation (A24) is independent of the roots  $\bar{m}_5$  and  $\bar{m}_6$ . It is mathematically the same as for a panel which is rigid in shear except that the roots  $\bar{m}_j$  are functions of the shear stiffness. (See eqs. (A11), (A10), and (A8).) Expanding equation (A24) and replacing the roots  $\bar{m}_j$  with the expressions given by equations (A11) yields.

$$\begin{aligned} F(\alpha, \delta, \epsilon, \gamma) = & \left[ (\delta^2 + \epsilon^2)^2 + 4\alpha^2(\delta^2 - \epsilon^2) + 4\gamma^2(4\alpha^2 + \delta^2 - \epsilon^2) \right] \sin \delta \sinh \epsilon \\ & - 8\delta\epsilon(\alpha^2 - \gamma^2)(\cosh \epsilon \cos \delta - \cosh 2\alpha) = 0 \end{aligned} \quad (A25)$$

Equation (A25) reduces to equation (9) of reference 6 when  $\gamma$  equals zero (infinite transverse shear stiffness).

Relations (A10), (A13), (A14), and (A25) are 8 independent equations involving 13 quantities ( $r, k_x, \eta, \phi, n, \lambda, \bar{\lambda}, \bar{A}, \bar{B}, \gamma, \alpha, \delta, \epsilon$ ). For simple supports these equations constitute the solution to equations (A4) once 5 of the 13 quantities are specified.

The differences between the results of the analysis presented herein and the results of reference 6 are due to the parameter  $r$ . In figure 11, some of these differences are illustrated by showing the relation between the frequency loops and the functions  $\bar{A}$  and  $\bar{B}$  which are defined by equations (A10). Regardless of  $r$ ,  $\lambda$  reaches the value  $\lambda_{cr}$  when two of the frequencies of the panel coalesce. An increase in  $\lambda$  above  $\lambda_{cr}$  causes  $\omega^2$  to become complex. Thus, one of the two square roots of  $\omega^2$  must possess a negative imaginary part which by equation (A6) is associated with divergent motion or flutter.

When  $r$  is zero (infinite shear stiffness), the expressions for  $\bar{A}$  and  $\bar{B}$  reduce to the definitions in reference 6. In this case  $\bar{A}$  is not a function of the frequency parameter  $\phi$  and depends only on  $n, \eta$ , and  $k_x$ . The frequency loops (variation of  $\lambda$  with  $\phi$ ) then lie in planes of constant  $\bar{A}$  and  $\lambda_{cr}$  is a function of  $\bar{A}$  only. Hence, differing panel configurations with the same value of  $\bar{A}$  but with different combinations of  $\eta$  and  $k_x$  will all theoretically flutter at an identical value of  $\lambda_{cr}$ . For  $r$  greater

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than zero (finite shear stiffness),  $\bar{A}$  is no longer independent of  $\phi$ . As a result, the frequency loops are rotated out of the constant  $\bar{A}$  planes by the angle

$$\Omega = \tan^{-1} \frac{r/\eta^2}{1+r} \quad (A26)$$

In this case  $n$ ,  $\eta$ , and  $k_X$  do not form a single parameter determining  $\lambda_{cr}$ .

Two additional effects of nonzero  $r$  can be seen from figure 11. One of these is that the frequency loop tends to bend over in the direction of increasing frequency. As  $r$  increases, this effect becomes more pronounced. The second feature is that for  $\lambda = 0$  (no airflow), the natural frequencies of the panel are reduced in magnitude as  $r$  is increased.

### Procedure for Obtaining Numerical Values From Exact Solution

Since the flutter value of  $\lambda$  is sought for a given panel configuration and midplane stress condition, it is desirable to specify  $r$ ,  $k_X$ , and  $\eta$ . If  $n$  and the frequency parameter  $\phi$  are then prescribed, the remaining eight quantities can be determined. This process enables one to obtain the variation of  $\lambda$  with  $\phi$  for fixed values of the length-width ratio and stress and shear-flexibility parameters.

A practical procedure for obtaining flutter values of  $\lambda$  for selected values of  $r$ ,  $k_X$ ,  $\eta$ , and  $n$  is as follows:

(1) Select  $\phi$ , a good choice being a value midway between the first and second invacuo values of  $\phi$  (see appendix C).

(2) Calculate  $\bar{A}$  and  $\bar{B}$  from equations (A10).

(3) Make an initial estimate for the correct value of  $\bar{\lambda}$ . Calculate corresponding values of  $\lambda$  and  $\gamma$  from equations (A10).

(4) Solve equation (A14) for  $\alpha^2$ . The Newton-Raphson technique works very well since only one real root of equation (A14) was found to exist when the estimated value of  $\bar{\lambda}$  was close to the correct value.

(5) Calculate  $\delta$  and  $\epsilon$  from equations (A13).

(6) Use equation (A25) to calculate  $F(\alpha, \delta, \epsilon, \gamma)$ , a nonzero value meaning an incorrect choice of  $\bar{\lambda}$ .

(7) Repeat steps 3 to 6 until a value of  $\bar{\lambda}$  is found that differs from the correct value [that is,  $F(\alpha, \delta, \epsilon, \gamma) = 0$ ] by only an acceptable amount. (An allowable error in  $\lambda$  of 0.01 percent was used in calculating the results presented herein.)

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(8) Increase  $\phi$  and repeat the process until the point  $\frac{\partial \lambda}{\partial \phi} = 0$  is obtained.

When the numerical results presented in this report were obtained, the increments in  $\phi$  were taken small enough to assure that the values of  $\lambda_{cr}$  were in error by no more than 1 percent. In most cases they are correct to 0.1 percent. In all cases, calculations show that the critical flutter solution is obtained by setting  $n = 1$  and determining  $\lambda_{cr}$  from the coalescence of the two lowest frequencies.

### Panel Mode Shape

The lateral deflection can be obtained from the first of equations (A6) once the coefficients  $A_j$  are known. These coefficients can be determined from equation (A23). If, in equation (A23),  $n$  is set equal to 1 and rows 5 and 6 are multiplied by  $r$ , and row 5 is subtracted from row 3 and row 6 is subtracted from row 4, the coefficients of  $B_5$  and  $B_6$  appearing in rows 3 and 4 can be made equal to zero. Then, replacing  $B_j$  with  $\frac{\pi^3 D}{b^3} \psi_j \bar{\beta}_j A_j$  for  $j = 1, 2, 3$ , or 4, where

$$\psi_j = \left( \frac{1}{1 - \bar{\beta}_j^2} + r \right)^{-1} \quad (A27)$$

and recalling that  $A_5 = A_6 = 0$  yields

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ e^{\bar{m}_1} & e^{\bar{m}_2} & e^{\bar{m}_3} & e^{\bar{m}_4} \\ \psi_1 & \psi_2 & \psi_3 & \psi_4 \\ \psi_1 e^{\bar{m}_1} & \psi_2 e^{\bar{m}_2} & \psi_3 e^{\bar{m}_3} & \psi_4 e^{\bar{m}_4} \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad (A28)$$

By a corollary to Cramer's rule (ref. 9), the coefficients  $A_j$  can now be determined within an arbitrary constant. Substituting the expressions for  $A_j$  and the expressions for  $\bar{m}_j$  given by equations (A11) into equation (A6) and ignoring the arbitrary constant yields, after considerable manipulation,

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$$\begin{aligned}
 w\left(\frac{x}{a}\right) = & \left[ -Te^{\xi} \sinh \epsilon + (\psi_3 - \psi_4)e^{\xi} \sin \delta \right] \left[ e^{\frac{\xi x}{a}} \cos \delta \frac{x}{a} - e^{\frac{\xi x}{a}} \cosh \epsilon \frac{x}{a} \right] \\
 & + \left[ Se^{\xi} \sinh \epsilon + (\psi_3 - \psi_4)e^{\xi} \cos \delta + \psi_3 e^{\xi - \epsilon} - \psi_4 e^{\xi + \epsilon} \right] e^{\frac{\xi x}{a}} \sin \delta \frac{x}{a} \\
 & + \left[ -Te^{\xi} \cosh \epsilon + (\psi_3 + \psi_4)e^{\xi} \sin \delta + e^{\xi} (T \cos \delta - S \sin \delta) \right] e^{\frac{\xi x}{a}} \sinh \epsilon \frac{x}{a} \quad (A29)
 \end{aligned}$$

where

$$\begin{aligned}
 \xi &= \gamma + \alpha \\
 \xi &= \gamma - \alpha \\
 \psi_3 &= \left[ \frac{1}{1 - \left( \frac{\xi + \epsilon}{\pi \eta} \right)^2} + r \right]^{-1} \\
 \psi_4 &= \left[ \frac{1}{1 - \left( \frac{\xi - \epsilon}{\pi \eta} \right)^2} + r \right]^{-1} \\
 \frac{S}{2} &= \frac{(1+r)\pi^4 \eta^4 + r(\xi^2 + \delta^2)^2 - \pi^2 \eta^2 (1+2r)(\xi^2 - \delta^2)}{(1+r)^2 \pi^4 \eta^4 + r^2(\xi^2 + \delta^2)^2 - 2\pi^2 \eta^2 r(1+r)(\xi^2 - \delta^2)} \\
 T &= \frac{-4\pi^2 \eta^2 \xi \delta}{(1+r)^2 \pi^4 \eta^4 + r^2(\xi^2 + \delta^2)^2 - 2\pi^2 \eta^2 r(1+r)(\xi^2 - \delta^2)} \quad (A30)
 \end{aligned}$$

The flutter mode shapes for  $w$  presented in figure 8 were obtained from equation (A29) by using the values of  $\alpha_{cr}$  in tables I and II.



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### PREFLUTTER SOLUTION

From equation (A25) it is seen that the transcendental equation  $F(\alpha, \delta, \epsilon, \gamma) = 0$  is satisfied identically when  $\epsilon = 2\alpha$  and  $\delta = 2m\pi$  where  $m$  is an integer. (This same relationship was noticed in reference 7 for panels which are rigid in shear, and leads to the so-called "preflutter" solution.) Substituting these expressions for  $\epsilon$  and  $\delta$  into equations (A12) and setting  $m = 1$  yields the following expression for  $\bar{\lambda}$

$$\frac{\bar{\lambda}}{\pi^3} = \frac{4}{3} \left[ 10 - \bar{A} + 6 \left( \frac{\gamma}{\pi} \right)^2 \right] \sqrt{\frac{4 - \bar{A}}{6} + \left( \frac{\gamma}{\pi} \right)^2} - 2 \left( \frac{\gamma}{\pi} \right) \bar{A} + 8 \left( \frac{\gamma}{\pi} \right)^3 \quad (B1)$$

(Setting  $m = 1$  restricts the preflutter solution to a point on the first frequency loop.) If the panel is rigid in shear ( $r = \gamma = 0$ ), the following preflutter equations for  $\lambda$  and  $\bar{B}$  are obtained

$$\left. \begin{aligned} \lambda &= \frac{4}{3} \left( \frac{\pi}{\eta} \right)^3 \left( 10 - \bar{A} \right) \sqrt{\frac{4 - \bar{A}}{6}} \\ \bar{B} &= \frac{4}{3} \left( 7 - 2\bar{A} \right) + \frac{\bar{A}^2}{12} \end{aligned} \right\} \quad (B2)$$

where  $\bar{A} = \eta^2(k_x - 2)$  is now independent of the panel frequency. As noted in reference 5, the first of equations (B2) yields values of  $\lambda$  that are very close to  $\lambda_{cr}$  when  $\bar{A}$  is negative. (See fig. 9.)

For  $r \neq 0$ , preflutter values of  $\lambda$  are not so easily obtained since it is not possible to solve for  $\lambda$  as an explicit function of  $r$ ,  $\eta$ , and  $k_x$ . However, it is possible to write equation (B1) in terms of  $r$ ,  $\eta$ , and  $\bar{A}$  by replacing  $\frac{\gamma}{\pi}$  with  $\frac{\bar{\lambda}}{4\pi^3} \left( \frac{r}{1+r} \right)^2$ . (See eqs. (A10).) The result is

$$\left( \frac{\bar{\lambda}}{\pi^3} \right)^2 = \frac{\left[ 1 + R\bar{A} - R^2 \frac{(\bar{A} - 60)(\bar{A} - 4)}{12} \right] \pm \sqrt{\left[ 1 + R\bar{A} - R^2 \frac{(\bar{A} - 60)(\bar{A} - 4)}{12} \right]^2 - \left( \frac{2}{3}R \right)^3 (1 + 4R)(10 - \bar{A})^2 (4 - \bar{A})}}{\frac{R^3}{2} (1 + 4R)} \quad (B3)$$

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where

$$R = \frac{r/\eta^2}{1+r}$$

Thus, if  $r$ ,  $\eta$ , and  $\bar{A}$  are specified,  $\bar{\lambda}$  can be calculated. (Note that  $\bar{A}$  is a function of the unknown frequency parameter  $\phi$ .) Since  $\bar{\lambda}$  is known,  $\frac{\gamma}{\pi}$  can be determined from equations (A10); and then  $\frac{\alpha}{\pi}$  and  $\bar{B}$  can be obtained from equations (A12). The stress parameter  $k_X$  is obtained by eliminating  $\phi$  from the expressions for  $\bar{A}$  and  $\bar{B}$  given by equations (A10):

$$k_X = \frac{\eta^2 \frac{2+r}{1+r} + (\bar{A} - R\bar{B})}{\eta^2(1+r) + r(\bar{A} - R\bar{B})} \quad (B4)$$

Finally,  $\lambda$  is determined from the third of equations (A10). It should be noted that for fixed  $r$  and  $\eta$ ,  $k_X$  cannot be specified arbitrarily since it is determined by the choice of  $\bar{A}$  used in equation (B3). Thus, trial-and-error calculations involving different choices of  $\bar{A}$  are necessary to obtain  $\lambda$  for the  $k_X$  of interest.

The preceding preflutter equations can be used for infinitely long panels by letting  $\eta$  approach  $\infty$ . This condition leads to the following expressions for  $\lambda$  and  $k_X$ :

$$\left. \begin{aligned} \lambda &= \sqrt{2}\pi^3 \Gamma \frac{1 - rk_X}{1+r} \\ k_X &= \frac{\frac{2+r}{1+r} + \left[ \left( \frac{\bar{A}}{\eta^2} \right) - P\left( \frac{\bar{B}}{\eta^4} \right)_\infty \right]}{1+r + r \left[ \left( \frac{\bar{A}}{\eta^2} \right) - P\left( \frac{\bar{B}}{\eta^4} \right)_\infty \right]} \end{aligned} \right\} \quad (B5)$$

## APPENDIX B

where

$$\left. \begin{aligned} \left(\frac{B}{\eta^4}\right)_{\infty} &= \frac{(P\Gamma)^4}{8} - \frac{(P\Gamma)^2}{4} \left(\frac{\bar{A}}{\eta^2}\right) + 2\sqrt{2}(P\Gamma) \left[ \frac{(P\Gamma)^2}{8} - \frac{1}{6} \left(\frac{\bar{A}}{\eta^2}\right) \right]^{3/2} + \frac{1}{12} \left(\frac{\bar{A}}{\eta^2}\right)^2 \\ \Gamma &= \left(\frac{\bar{\lambda}}{2\eta^6\pi^6}\right)_{\infty} = \frac{(12 + 12s - s^2) \pm \sqrt{144 + 288s + 120s^2 + \frac{56}{3}s^3 + s^4}}{12P^3} \\ P &= \frac{r}{1+r} \\ s &= P \frac{\bar{A}}{\eta^2} \end{aligned} \right\} \quad (B6)$$

Thus, by specifying  $r$  and  $\frac{\bar{A}}{\eta^2}$ ,  $\bar{\lambda}$  and  $k_X$  can be calculated for  $\eta = \infty$ .

An interesting solution occurs when  $\frac{\bar{A}}{\eta^2} = 0$ . If the negative root is used to calculate  $\Gamma$ ,  $\Gamma = \left(\frac{\bar{B}}{\eta^4}\right)_{\infty} = 0$ . Hence,

$$\left. \begin{aligned} k_X &= \frac{2+r}{(1+r)^2} \\ \lambda &= 0 \end{aligned} \right\} \quad (B7)$$

That is, as  $k_X$  is increased to the value  $\frac{2+r}{(1+r)^2}$ , the preflutter value of  $\lambda$  for an infinitely long panel drops to zero. It is shown in appendix C that  $\lambda_{cr}$  also goes to zero as  $k_X$  approaches  $\frac{2+r}{(1+r)^2}$  when  $\eta = \infty$ .

## APPENDIX C

### PANEL BEHAVIOR FOR ZERO DYNAMIC PRESSURE

The natural vibration characteristics of the panel can be obtained for zero dynamic pressure by setting  $\lambda = \bar{\lambda} = \gamma = 0$ . Then by equation (A14),  $\alpha$  is also equal to zero. Equation (A25) reduces to  $(\epsilon^2 + \delta^2)^2 \sin \delta \sinh \epsilon = 0$  so that

$$\delta = m\pi \quad (C1)$$

where  $m$  is an integer designating the number of sinusoidal half-waves in the  $x$ -direction. From equations (A12) and (C1), it is readily verified that  $\epsilon^2 = \frac{\pi^2 \bar{B}}{m^2}$  and

$$B = m^2(m^2 - \bar{A}) \quad (C2)$$

Equations (C2) and (A10) lead to

$$\phi = \frac{\left[\left(\frac{m}{\eta}\right)^2 + n^2\right]^2}{1 + r\left[\left(\frac{m}{\eta}\right)^2 + n^2\right]} - \left(\frac{m}{\eta}\right)^2 k_x \quad (C3)$$

Use of the expression for  $\phi$  as given by equation (A8) enables the in-vacuo natural frequencies to be expressed as

$$\left(\frac{\omega}{\omega_r}\right)^2 = \frac{\left[\left(\frac{m}{\eta}\right)^2 + n^2\right]^2}{1 + r\left[\left(\frac{m}{\eta}\right)^2 + n^2\right]} - \left[\left(\frac{m}{\eta}\right)^2 + n^2 \frac{N_y}{N_x}\right] k_x \quad (C4)$$

Equation (C4) can be used to calculate the in-vacuo buckling loads by equating the frequency to zero.

In the analysis presented in appendix A, it is assumed that flutter occurs when two frequencies of the panel coalesce or become equal. For certain values of the stress parameter  $k_x$ , two of the panel's in-vacuo natural frequencies become equal. This condition occurs whenever two consecutive mode lines cross one another as in figure 11 (that is, eq. (C2) is satisfied by two consecutive integer values of  $m$ ). The numerical calculations which were made indicate that  $\lambda_{cr} = 0$  at these points; that is, the frequency

## APPENDIX C

loop degenerates to a point. The combinations of  $r$ ,  $k_x$ , and  $\eta$  which produce these theoretical zero-dynamic-pressure flutter points can be determined by using equation (C3) to eliminate  $\phi$  from the expression for  $\bar{B}$  given by the last of equations (A10). Using the in-vacuo relation between  $\bar{B}$  and  $\bar{A}$  (eq. (C2)) then leads to

$$\eta^4 \left[ k_x - \frac{2+r}{(1+r)^2} \right] - \eta^2 \left( \bar{A} \frac{1-rk_x}{1+r} \right) + r \bar{B} \frac{1-rk_x}{1+r} = 0 \quad (C5)$$

This equation gives the zero-dynamic-pressure flutter points in terms of  $r$ ,  $k_x$ , and  $\eta$ . The values of  $\bar{A}$  and  $\bar{B}$  to be used are those for which equation (C2) is satisfied by two consecutive integer values of  $m$ . (The first zero-dynamic-pressure flutter point occurs at  $\bar{A} = 5$ .) (See fig. 6.)

## REFERENCES

1. Fung, Y. C.: Some Recent Contributions to Panel Flutter Research. AIAA J., vol. 1, no. 4, Apr. 1963, pp. 898-909.
2. McElman, John A.: Flutter of Curved and Flat Sandwich Panels Subjected to Supersonic Flow. NASA TN D-2192, 1964.
3. Weidman, Deene J.: Effects of Side-Edge Boundary Conditions and Transverse Shear Stiffnesses on the Flutter of Orthotropic Panels in Supersonic Flow. NASA TN D-3302, 1966.
4. Libove, Charles; and Batdorf, S. B.: A General Small-Deflection Theory for Flat Sandwich Plates. NACA Rept. 899, 1948. (Supersedes NACA TN 1526.)
5. Bohon, Herman L.; and Dixon, Sidney C.: Some Recent Developments in Flutter of Flat Panels. J. Aircraft, vol. 1, no. 5, Sept.-Oct. 1964, pp. 280-288.
6. Hedgepeth, John M.: Flutter of Rectangular Simply Supported Panels at High Supersonic Speeds. J. Aeron. Sci., vol. 24, no. 8, Aug. 1957, pp. 563-573, 586.
7. Movchan, A. A.: On the Stability of a Panel Moving in a Gas. NASA RE 11-21-58W, 1959.
8. Bijlaard, P. P.: Stability of Sandwich Plates. J. Aeron. Sci. (Readers' Forum), vol. 16, no. 9, Sept. 1949, pp. 573-574.
9. Wylie, C. R., Jr.: Advanced Engineering Mathematics. Second ed., McGraw-Hill Book Co., Inc., 1960, p. 34.

TABLE I - FLUTTER SOLUTIONS FOR PANELS WITH LENGTH-WIDTH RATIOS LESS THAN ONE

| a/b | $\lambda'_{cr}$ | $\phi'_{cr}$ | $\alpha_{cr}$ | $\lambda'_{cr}$ | $\phi'_{cr}$ | $\alpha_{cr}$ | $\lambda'_{cr}$ | $\phi'_{cr}$ | $\alpha_{cr}$ | $\lambda'_{cr}$ | $\phi'_{cr}$ | $\alpha_{cr}$ | r'        |
|-----|-----------------|--------------|---------------|-----------------|--------------|---------------|-----------------|--------------|---------------|-----------------|--------------|---------------|-----------|
|     | $k'_X = -4$     |              |               | $k'_X = -2$     |              |               | $k'_X = -1$     |              |               | $k'_X = 0$      |              |               |           |
| 0.0 | 696.5           | 25.25        | 3.495         | 512.5           | 17.70        | 3.024         | 425.8           | 14.00        | 2.774         | 343.4           | 10.79        | 2.477         | 0.00<br>↓ |
| .2  | 704.0           | 25.51        | 3.516         | 519.5           | 17.95        | 3.047         | 432.8           | 14.55        | 2.777         | 349.8           | 10.99        | 2.506         |           |
| .4  | 727.9           | 27.01        | 3.540         | 541.1           | 19.08        | 3.091         | 453.3           | 15.62        | 2.826         | 369.3           | 12.12        | 2.552         |           |
| .6  | 766.9           | 29.18        | 3.600         | 577.4           | 20.87        | 3.176         | 488.0           | 16.96        | 2.944         | 402.0           | 13.00        | 2.708         |           |
| .8  | 822.3           | 31.36        | 3.742         | 629.0           | 23.32        | 3.305         | 537.6           | 19.34        | 3.080         | 449.8           | 15.88        | 2.814         |           |
|     | $k'_X = -4$     |              |               | $k'_X = -2$     |              |               | $k'_X = -1$     |              |               | $k'_X = 0$      |              |               |           |
| 0.0 | 707.5           | 28.11        | 3.168         | 496.8           | 18.27        | 2.859         | 398.5           | 13.74        | 2.668         | 306.0           | 9.929        | 2.407         | 0.05<br>↓ |
| .2  | 715.4           | 27.74        | 3.228         | 503.1           | 18.52        | 2.881         | 404.3           | 14.47        | 2.652         | 311.3           | 10.13        | 2.433         |           |
| .4  | 737.0           | 29.07        | 3.259         | 522.0           | 19.15        | 2.953         | 422.1           | 15.12        | 2.727         | 327.6           | 11.06        | 2.483         |           |
| .6  | 772.6           | 30.31        | 3.368         | 553.9           | 21.02        | 3.021         | 451.6           | 16.51        | 2.826         | 354.8           | 12.49        | 2.581         |           |
| .8  | 823.6           | 33.54        | 3.445         | 598.5           | 23.38        | 3.138         | 493.0           | 18.48        | 2.969         | 393.2           | 14.48        | 2.723         |           |
|     | $k'_X = -4$     |              |               | $k'_X = -2$     |              |               | $k'_X = -1$     |              |               | $k'_X = 0$      |              |               |           |
| 0.0 | 752.6           | 32.91        | 3.094         | 478.8           | 19.72        | 2.816         | 352.9           | 13.63        | 2.643         | 236.8           | 8.486        | 2.383         | 0.20<br>↓ |
| .2  | 759.4           | 33.58        | 3.103         | 483.9           | 19.85        | 2.845         | 357.4           | 13.96        | 2.659         | 240.5           | 8.682        | 2.407         |           |
| .4  | 779.1           | 34.61        | 3.166         | 499.4           | 21.03        | 2.889         | 370.7           | 14.77        | 2.717         | 251.5           | 9.237        | 2.482         |           |
| .6  | 811.6           | 36.92        | 3.247         | 524.8           | 22.56        | 2.986         | 392.4           | 16.14        | 2.813         | 269.5           | 10.37        | 2.585         |           |
| .8  | 855.8           | 39.96        | 3.367         | 559.3           | 24.77        | 3.118         | 422.1           | 18.00        | 2.952         | 294.2           | 12.08        | 2.718         |           |
|     | $k'_X = -4$     |              |               | $k'_X = -2$     |              |               | $k'_X = -1$     |              |               | $k'_X = 0$      |              |               |           |
| 0.0 | 811.4           | 40.47        | 3.445         | 478.8           | 22.16        | 3.104         | 326.3           | 14.37        | 2.860         | 187.1           | 7.242        | 2.549         | 0.40<br>↓ |
| .2  | 817.9           | 40.59        | 3.475         | 483.3           | 22.65        | 3.122         | 330.0           | 14.49        | 2.892         | 189.8           | 7.610        | 2.557         |           |
| .4  | 836.7           | 41.83        | 3.547         | 497.0           | 23.37        | 3.203         | 340.8           | 15.21        | 2.967         | 197.7           | 8.030        | 2.645         |           |
| .6  | 868.2           | 44.65        | 3.658         | 518.9           | 25.11        | 3.316         | 358.0           | 16.51        | 3.083         | 210.3           | 8.990        | 2.763         |           |
| .8  | 909.1           | 48.62        | 3.802         | 548.3           | 27.48        | 3.470         | 381.1           | 18.37        | 3.238         | 227.0           | 10.20        | 2.934         |           |
|     | $k'_X = -4$     |              |               | $k'_X = -2$     |              |               | $k'_X = -1$     |              |               | $k'_X = 0$      |              |               |           |
| 0.0 | 947.3           | 57.87        | 4.814         | 507.8           | 28.95        | 4.319         | 304.9           | 16.33        | 3.916         | 121.8           | 5.720        | 3.221         | 1.00<br>↓ |
| .2  | 953.5           | 58.22        | 4.846         | 511.8           | 29.38        | 4.352         | 307.6           | 16.58        | 3.951         | 123.2           | 5.856        | 3.255         |           |
| .4  | 972.8           | 60.60        | 4.943         | 523.5           | 30.42        | 4.451         | 315.6           | 17.29        | 4.050         | 127.4           | 6.221        | 3.358         |           |
| .6  | 1003            | 63.84        | 5.095         | 542.1           | 32.31        | 4.606         | 328.3           | 18.59        | 4.208         | 134.0           | 6.816        | 3.521         |           |
| .8  | 1042            | 67.91        | 5.292         | 566.3           | 34.84        | 4.808         | 344.8           | 20.28        | 4.413         | 142.6           | 7.689        | 3.730         |           |
|     | $k'_X = -4$     |              |               | $k'_X = -2$     |              |               | $k'_X = -1$     |              |               | $k'_X = 0$      |              |               |           |
| 0.0 | 1093            | 79.02        | 6.154         | 556.8           | 37.78        | 5.644         | 306.6           | 19.56        | 5.183         | 81.14           | 4.431        | 4.135         | 2.00<br>↓ |
| .2  | 1100            | 79.92        | 6.191         | 560.5           | 38.25        | 5.682         | 308.9           | 19.88        | 5.222         | 81.96           | 4.522        | 4.176         |           |
| .4  | 1119            | 82.42        | 6.299         | 571.3           | 39.66        | 5.791         | 315.6           | 20.55        | 5.334         | 84.28           | 4.808        | 4.292         |           |
| .6  | 1148            | 86.59        | 6.465         | 588.4           | 41.69        | 5.964         | 326.0           | 21.88        | 5.510         | 87.98           | 5.189        | 4.479         |           |
| .8  | 1187            | 91.65        | 6.682         | 610.1           | 44.52        | 6.183         | 339.3           | 23.53        | 5.734         | 92.65           | 5.759        | 4.714         |           |

TABLE I.- FLUTTER SOLUTIONS FOR PANELS WITH LENGTH-WIDTH RATIOS LESS THAN ONE - Concluded

| a/b | $\lambda'_{cr}$ | $\phi'_{cr}$ | $\alpha_{cr}$ | $\lambda'_{cr}$ | $\phi'_{cr}$ | $\alpha_{cr}$ | $\lambda'_{cr}$ | $\phi'_{cr}$ | $\alpha_{cr}$ | $\lambda'_{cr}$ | $\phi'_{cr}$ | $\alpha_{cr}$ | r'   |
|-----|-----------------|--------------|---------------|-----------------|--------------|---------------|-----------------|--------------|---------------|-----------------|--------------|---------------|------|
|     | $k'_x = 1$      |              |               | $k'_x = 2$      |              |               | $k'_x = 3$      |              |               | $k'_x = 4$      |              |               |      |
| 0.0 | 264.9           | 7.400        | 2.174         | 190.9           | 4.400        | 1.807         | 121.8           | 1.350        | 1.390         | 57.98           | -1.400       | 0.8248        | 0.00 |
| .2  | 271.0           | 7.834        | 2.185         | 196.7           | 4.626        | 1.840         | 127.2           | 1.618        | 1.424         | 62.88           | -1.190       | .8785         |      |
| .4  | 289.5           | 8.554        | 2.270         | 214.1           | 5.338        | 1.934         | 143.4           | 2.348        | 1.532         | 77.80           | -.5094       | 1.027         |      |
| .6  | 320.9           | 9.818        | 2.406         | 243.6           | 6.546        | 2.088         | 171.1           | 3.616        | 1.703         | 103.4           | +.6876       | 1.250         |      |
| .8  | 366.1           | 12.25        | 2.550         | 286.4           | 8.822        | 2.256         | 211.2           | 5.610        | 1.918         | 140.7           | 2.566        | 1.520         |      |
|     | $k'_x = 0.9$    |              |               | $k'_x = 1.859$  |              |               | $k'_x = 2.7$    |              |               | $k'_x = 3.6$    |              |               |      |
| 0.0 | 227.8           | 6.506        | 2.144         | 150.9           | 3.401        | 1.760         | 89.33           | 0.7496       | 1.334         | 30.50           | -1.804       | 0.6333        | 0.05 |
| .2  | 232.9           | 6.859        | 2.157         | 155.4           | 3.585        | 1.794         | 93.34           | .9243        | 1.375         | 33.97           | -1.637       | .6937         |      |
| .4  | 247.8           | 7.620        | 2.224         | 168.8           | 4.272        | 1.876         | 105.4           | 1.544        | 1.481         | 44.47           | -1.107       | .8617         |      |
| .6  | 272.8           | 8.655        | 2.362         | 191.5           | 5.404        | 2.015         | 125.8           | 2.497        | 1.663         | 62.33           | -.1511       | 1.106         |      |
| .8  | 308.3           | 10.80        | 2.491         | 223.7           | 7.057        | 2.203         | 155.0           | 4.149        | 1.872         | 87.99           | 1.330        | 1.393         |      |
|     | $k'_x = 0.7$    |              |               | $k'_x = 1.4$    |              |               | $k'_x = 1.528$  |              |               | $k'_x = 2.1$    |              |               |      |
| 0.0 | 162.5           | 5.096        | 2.155         | 95.58           | 2.261        | 1.798         | 84.27           | 1.736        | 1.723         | 38.05           | -0.1841      | 1.192         | 0.20 |
| .2  | 165.7           | 5.323        | 2.174         | 98.15           | 2.379        | 1.834         | 86.76           | 1.911        | 1.748         | 40.01           | -.0443       | 1.229         |      |
| .4  | 175.0           | 5.812        | 2.254         | 105.8           | 2.877        | 1.913         | 94.12           | 2.330        | 1.843         | 45.92           | .3000        | 1.361         |      |
| .6  | 190.4           | 6.805        | 2.365         | 118.5           | 3.655        | 2.053         | 106.3           | 3.152        | 1.975         | 55.69           | .9585        | 1.547         |      |
| .8  | 211.5           | 8.132        | 2.527         | 135.9           | 4.826        | 2.232         | 122.9           | 4.296        | 2.159         | 69.16           | 1.914        | 1.786         |      |
|     | $k'_x = 0.5$    |              |               | $k'_x = 1.0$    |              |               | $k'_x = 1.224$  |              |               | $k'_x = 1.5$    |              |               |      |
| 0.0 | 124.6           | 4.416        | 2.301         | 68.40           | 1.829        | 1.963         | 45.98           | 0.8732       | 1.725         | 21.62           | -0.1593      | 1.273         | 0.40 |
| .2  | 126.7           | 4.504        | 2.336         | 70.02           | 1.992        | 1.982         | 47.34           | .9808        | 1.756         | 22.67           | -.0860       | 1.322         |      |
| .4  | 133.0           | 4.972        | 2.412         | 74.76           | 2.324        | 2.074         | 51.36           | 1.281        | 1.855         | 25.78           | .1730        | 1.445         |      |
| .6  | 143.2           | 5.703        | 2.542         | 82.41           | 2.882        | 2.222         | 57.87           | 1.790        | 2.013         | 30.81           | .5998        | 1.642         |      |
| .8  | 156.7           | 6.818        | 2.706         | 92.67           | 3.733        | 2.407         | 66.60           | 2.566        | 2.207         | 37.62           | 1.270        | 1.871         |      |
|     | $k'_x = 0.25$   |              |               | $k'_x = 0.50$   |              |               | $k'_x = 0.75$   |              |               | $k'_x = 0.80$   |              |               |      |
| 0.0 | 81.17           | 3.526        | 2.934         | 44.12           | 1.637        | 2.530         | 12.88           | 0.1550       | 1.801         | 7.824           | -0.0650      | 1.518         | 1.00 |
| .2  | 82.27           | 3.642        | 2.968         | 44.88           | 1.702        | 2.567         | 13.28           | .2079        | 1.836         | 8.143           | -.0232       | 1.565         |      |
| .4  | 85.48           | 3.905        | 3.074         | 47.08           | 1.908        | 2.674         | 14.43           | .3378        | 1.964         | 9.074           | .1042        | 1.696         |      |
| .6  | 90.53           | 4.392        | 3.237         | 50.54           | 2.262        | 2.841         | 16.25           | .5816        | 2.143         | 10.54           | .3224        | 1.894         |      |
| .8  | 97.08           | 5.071        | 3.448         | 55.03           | 2.772        | 3.054         | 18.59           | .9270        | 2.373         | 12.43           | .6306        | 2.146         |      |
|     | $k'_x = 0.125$  |              |               | $k'_x = 0.25$   |              |               | $k'_x = 0.375$  |              |               | $k'_x = 0.4444$ |              |               |      |
| 0.0 | 56.42           | 2.918        | 3.848         | 33.28           | 1.576        | 3.436         | 12.68           | 0.4588       | 2.724         | 3.369           | 0.0000       | 1.879         | 2.00 |
| .2  | 57.05           | 2.994        | 3.889         | 33.71           | 1.622        | 3.479         | 12.90           | .4796        | 2.772         | 3.476           | .0217        | 1.929         |      |
| .4  | 58.85           | 3.187        | 4.010         | 34.95           | 1.769        | 3.602         | 13.55           | .5854        | 2.892         | 3.785           | .0889        | 2.068         |      |
| .6  | 61.65           | 3.518        | 4.197         | 36.86           | 2.007        | 3.792         | 14.57           | .7337        | 3.093         | 4.260           | .2030        | 2.277         |      |
| .8  | 65.21           | 3.963        | 4.434         | 39.29           | 2.323        | 4.034         | 15.84           | .9466        | 3.342         | 4.860           | .3643        | 2.538         |      |



TABLE II.- FLUTTER SOLUTIONS FOR PANELS WITH LENGTH-WIDTH RATIOS GREATER THAN OR EQUAL TO ONE

| a/b        | $\lambda_{cr}$ | $\phi_{cr}$ | $\alpha_{cr}$ | $\lambda_{cr}$ | $\phi_{cr}$ | $\alpha_{cr}$ | $\lambda_{cr}$ | $\phi_{cr}$ | $\alpha_{cr}$ | $\lambda_{cr}$ | $\phi_{cr}$ | $\alpha_{cr}$ | r    |
|------------|----------------|-------------|---------------|----------------|-------------|---------------|----------------|-------------|---------------|----------------|-------------|---------------|------|
|            | $k_x = -4$     |             |               | $k_x = -2$     |             |               | $k_x = -1$     |             |               | $k_x = 0$      |             |               |      |
| 1.00       | 895.4          | 35.23       | 3.877         | 697.0          | 27.06       | 3.448         | 603.0          | 23.00       | 3.229         | 512.5          | 19.22       | 2.988         | 0.0  |
| 1.25       | 644.5          | 21.59       | 4.481         | 471.6          | 15.58       | 3.911         | 391.0          | 12.85       | 3.592         | 314.7          | 10.18       | 3.263         |      |
| 1.50       | 517.9          | 15.38       | 5.117         | 360.5          | 10.61       | 4.405         | 288.3          | 8.466       | 4.009         | 220.8          | 6.449       | 3.586         |      |
| 1.75       | 444.6          | 12.04       | 5.767         | 297.6          | 7.986       | 4.938         | 230.7          | 6.170       | 4.486         | 169.5          | 4.739       | 3.884         |      |
| 2.00       | 398.3          | 9.906       | 6.497         | 258.3          | 6.475       | 5.477         | 195.6          | 5.016       | 4.895         | 138.3          | 3.726       | 4.238         |      |
| 2.50       | 344.6          | 7.616       | 7.965         | 212.6          | 4.708       | 6.747         | 155.8          | 3.694       | 5.847         | 104.0          | 2.724       | 4.958         |      |
| 3.00       | 315.7          | 6.486       | 9.408         | 189.7          | 4.023       | 7.793         | 134.9          | 3.035       | 6.873         | 86.38          | 2.237       | 5.751         |      |
| 4.00       | 286.7          | 5.375       | 12.43         | 166.2          | 3.259       | 10.21         | 114.4          | 2.434       | 9.001         | 69.39          | 1.801       | 7.461         |      |
| 5.00       | 273.2          | 4.874       | 15.50         | 155.3          | 2.920       | 12.67         | 105.1          | 2.187       | 11.08         | 61.75          | 1.629       | 9.118         |      |
| 7.00       | 261.1          | 4.444       | 21.70         | 145.6          | 2.630       | 17.71         | 96.75          | 1.974       | 15.32         | 54.98          | 1.479       | 12.68         |      |
| 10.00      | 254.6          | 4.218       | 31.04         | 140.3          | 2.479       | 25.32         | 92.23          | 1.859       | 21.92         | 51.39          | 1.406       | 17.89         |      |
| 15.00      | 251.0          | 4.097       | 46.69         | 137.4          | 2.398       | 38.09         | 89.75          | 1.799       | 32.93         | 49.40          | 1.366       | 26.81         |      |
| 20.00      | 249.7          | 4.054       | 62.46         | 136.4          | 2.370       | 50.89         | 88.87          | 1.777       | 44.04         | 48.68          | 1.351       | 35.89         |      |
| 50.00*     | 248.2          | 4.006       | 157.1         | 135.2          | 2.338       | 128.3         | 87.84          | 1.753       | 111.1         | 47.85          | 1.335       | 90.73         |      |
| 100.00*    | 248.1          | 4.002       | 314.2         | 135.1          | 2.334       | 256.5         | 87.73          | 1.751       | 222.2         | 47.77          | 1.334       | 181.4         |      |
| $\infty^*$ | 248.0          | 4.000       | $\infty$      | 135.0          | 2.333       | $\infty$      | 87.70          | 1.750       | $\infty$      | 47.73          | 1.333       | $\infty$      |      |
|            | $k_x = -4$     |             |               | $k_x = -2$     |             |               | $k_x = -1$     |             |               | $k_x = 0$      |             |               |      |
| 1.00       | 888.5          | 37.42       | 3.565         | 656.0          | 26.70       | 3.279         | 546.8          | 21.74       | 3.108         | 442.9          | 17.14       | 2.907         | 0.05 |
| 1.25       | 652.0          | 22.91       | 4.121         | 456.9          | 15.63       | 3.723         | 366.6          | 12.30       | 3.486         | 281.9          | 9.413       | 3.174         |      |
| 1.50       | 527.0          | 16.43       | 4.662         | 354.4          | 10.71       | 4.198         | 275.3          | 8.348       | 3.857         | 201.9          | 6.112       | 3.482         |      |
| 1.75       | 453.6          | 12.45       | 5.365         | 294.7          | 8.109       | 4.695         | 222.8          | 6.211       | 4.280         | 156.8          | 4.457       | 3.814         |      |
| 2.00       | 406.3          | 10.35       | 5.976         | 256.8          | 6.550       | 5.221         | 189.8          | 4.976       | 4.717         | 129.0          | 3.555       | 4.140         |      |
| 2.50       | 350.7          | 7.931       | 7.286         | 213.0          | 4.897       | 6.296         | 152.2          | 3.648       | 5.664         | 97.86          | 2.591       | 4.881         |      |
| 3.00       | 320.3          | 6.677       | 8.645         | 189.4          | 4.047       | 7.439         | 132.2          | 3.001       | 6.645         | 81.64          | 2.143       | 5.642         |      |
| 4.00       | 289.7          | 5.484       | 11.40         | 165.9          | 3.270       | 9.713         | 112.4          | 2.404       | 8.685         | 65.90          | 1.730       | 7.309         |      |
| 5.00       | 275.2          | 4.952       | 14.17         | 154.9          | 2.197       | 12.09         | 103.3          | 2.154       | 10.69         | 58.51          | 1.564       | 8.924         |      |
| 7.00       | 262.2          | 4.486       | 19.87         | 145.0          | 2.619       | 16.88         | 95.10          | 1.933       | 14.90         | 52.38          | 1.420       | 12.37         |      |
| 10.00      | 255.2          | 4.238       | 28.43         | 139.6          | 2.462       | 24.13         | 90.64          | 1.818       | 21.27         | 48.94          | 1.347       | 17.60         |      |
| 15.00      | 251.3          | 4.107       | 42.84         | 136.7          | 2.378       | 36.29         | 88.19          | 1.757       | 32.00         | 47.05          | 1.308       | 26.43         |      |
| 20.00      | 249.8          | 4.059       | 57.34         | 135.6          | 2.349       | 48.50         | 87.31          | 1.736       | 42.74         | 46.37          | 1.294       | 35.32         |      |
| 50.00*     | 248.3          | 4.007       | 144.0         | 134.4          | 2.315       | 122.3         | 86.29          | 1.712       | 107.9         | 45.58          | 1.279       | 89.31         |      |
| 100.00*    | 248.1          | 4.002       | 288.0         | 134.3          | 2.312       | 244.6         | 86.19          | 1.709       | 215.8         | 45.50          | 1.277       | 178.6         |      |
| $\infty^*$ | 248.0          | 4.000       | $\infty$      | 134.2          | 2.311       | $\infty$      | 86.15          | 1.709       | $\infty$      | 45.47          | 1.277       | $\infty$      |      |
|            | $k_x = -4$     |             |               | $k_x = -2$     |             |               | $k_x = -1$     |             |               | $k_x = 0$      |             |               |      |
| 1.00       | 910.9          | 43.97       | 3.516         | 602.4          | 27.80       | 3.275         | 459.1          | 20.74       | 3.106         | 325.0          | 14.19       | 2.899         | 0.2  |
| 1.25       | 684.3          | 26.85       | 3.850         | 437.6          | 16.52       | 3.571         | 324.1          | 12.08       | 3.363         | 219.2          | 8.008       | 3.104         |      |
| 1.50       | 557.7          | 18.58       | 4.304         | 347.0          | 11.32       | 3.949         | 250.8          | 8.146       | 3.700         | 162.8          | 5.335       | 3.359         |      |
| 1.75       | 480.6          | 14.42       | 4.743         | 292.0          | 8.496       | 4.385         | 206.5          | 5.978       | 4.118         | 129.4          | 3.920       | 3.675         |      |
| 2.00       | 428.1          | 11.46       | 5.331         | 256.2          | 6.906       | 4.807         | 178.3          | 4.879       | 4.467         | 108.1          | 3.138       | 3.989         |      |
| 2.50       | 367.7          | 8.722       | 6.364         | 213.5          | 5.104       | 5.744         | 144.7          | 3.569       | 5.330         | 83.57          | 2.308       | 4.690         |      |
| 3.00       | 333.4          | 7.280       | 7.420         | 189.9          | 4.165       | 6.778         | 126.4          | 2.911       | 6.262         | 70.35          | 1.893       | 5.484         |      |
| 4.00       | 298.0          | 5.820       | 9.740         | 165.9          | 3.311       | 8.837         | 107.9          | 2.327       | 8.088         | 57.37          | 1.548       | 6.986         |      |
| 5.00       | 281.0          | 5.166       | 12.08         | 154.4          | 2.931       | 10.93         | 99.11          | 2.061       | 10.04         | 51.28          | 1.393       | 8.601         |      |
| 7.00       | 265.5          | 4.598       | 16.86         | 144.0          | 2.603       | 15.19         | 91.22          | 1.840       | 13.86         | 45.56          | 1.253       | 12.16         |      |
| 10.00      | 256.9          | 4.296       | 24.12         | 138.3          | 2.428       | 21.76         | 86.88          | 1.721       | 19.87         | 42.88          | 1.198       | 16.94         |      |
| 15.00      | 252.1          | 4.132       | 36.35         | 135.1          | 2.333       | 32.84         | 84.46          | 1.658       | 29.89         | 41.24          | 1.163       | 25.44         |      |
| 20.00      | 250.2          | 4.070       | 48.72         | 133.9          | 2.301       | 43.87         | 83.59          | 1.636       | 39.92         | 40.64          | 1.151       | 34.00         |      |
| 50.00*     | 248.3          | 4.009       | 122.2         | 132.6          | 2.263       | 110.4         | 82.57          | 1.611       | 100.9         | 39.95          | 1.136       | 86.02         |      |
| 100.00*    | 248.1          | 4.002       | 244.4         | 132.5          | 2.259       | 220.7         | 82.46          | 1.608       | 201.7         | 39.87          | 1.135       | 172.0         |      |
| $\infty^*$ | 248.0          | 4.000       | $\infty$      | 132.4          | 2.258       | $\infty$      | 82.43          | 1.607       | $\infty$      | 39.85          | 1.134       | $\infty$      |      |

\*Denotes preflutter solution for this value of a/b.

TABLE II.- FLUTTER SOLUTIONS FOR PANELS WITH LENGTH-WIDTH RATIOS GREATER THAN OR EQUAL TO ONE - Continued

| a/b        | $\lambda_{cr}$ | $\phi_{cr}$ | $\alpha_{cr}$ | $\lambda_{cr}$ | $\phi_{cr}$ | $\alpha_{cr}$ | $\lambda_{cr}$ | $\phi_{cr}$ | $\alpha_{cr}$ | $\lambda_{cr}$ | $\phi_{cr}$ | $\alpha_{cr}$ | r   |
|------------|----------------|-------------|---------------|----------------|-------------|---------------|----------------|-------------|---------------|----------------|-------------|---------------|-----|
|            | $k_X = -4$     |             |               | $k_X = -2$     |             |               | $k_X = -1$     |             |               | $k_X = 0$      |             |               |     |
| 1.00       | 960.0          | 52.41       | 3.999         | 583.9          | 30.52       | 3.659         | 409.2          | 20.69       | 3.431         | 247.5          | 12.03       | 3.122         | 0.4 |
| 1.25       | 729.1          | 31.89       | 4.075         | 433.6          | 18.13       | 3.751         | 297.6          | 12.12       | 3.526         | 172.9          | 6.939       | 3.194         |     |
| 1.50       | 596.0          | 21.89       | 4.330         | 348.1          | 12.38       | 3.991         | 234.6          | 8.182       | 3.759         | 131.6          | 4.646       | 3.386         |     |
| 1.75       | 512.6          | 16.57       | 4.667         | 294.9          | 9.253       | 4.318         | 195.8          | 6.106       | 4.057         | 106.4          | 3.453       | 3.639         |     |
| 2.00       | 456.7          | 13.41       | 5.042         | 259.4          | 7.437       | 4.667         | 170.0          | 4.863       | 4.405         | 89.97          | 2.754       | 3.939         |     |
| 2.50       | 388.1          | 9.828       | 5.912         | 216.4          | 5.402       | 5.489         | 139.1          | 3.528       | 5.181         | 70.49          | 2.012       | 4.628         |     |
| 3.00       | 349.0          | 7.999       | 6.842         | 192.2          | 4.374       | 6.372         | 121.9          | 2.888       | 5.961         | 59.89          | 1.673       | 5.315         |     |
| 4.00       | 308.0          | 6.238       | 8.816         | 167.0          | 3.415       | 8.190         | 104.1          | 2.260       | 7.718         | 49.12          | 1.354       | 6.838         |     |
| 5.00       | 288.0          | 5.434       | 10.89         | 154.8          | 2.978       | 10.12         | 95.58          | 1.989       | 9.492         | 44.06          | 1.223       | 8.327         |     |
| 7.00       | 269.5          | 4.744       | 15.10         | 143.6          | 2.603       | 14.06         | 87.80          | 1.754       | 13.16         | 39.47          | 1.107       | 11.50         |     |
| 10.00      | 259.0          | 4.370       | 21.50         | 137.3          | 2.405       | 20.04         | 83.44          | 1.631       | 18.74         | 36.92          | 1.048       | 16.33         |     |
| 15.00      | 253.1          | 4.168       | 32.28         | 133.8          | 2.297       | 30.10         | 80.99          | 1.564       | 28.15         | 35.50          | 1.016       | 24.58         |     |
| 20.00      | 250.8          | 4.088       | 43.41         | 132.4          | 2.257       | 40.51         | 80.11          | 1.541       | 37.62         | 34.98          | 1.004       | 32.84         |     |
| 50.00*     | 248.4          | 4.011       | 108.8         | 131.0          | 2.216       | 101.5         | 79.06          | 1.513       | 95.05         | 34.37          | .9915       | 83.09         |     |
| 100.00*    | 248.1          | 4.002       | 217.5         | 130.8          | 2.211       | 203.0         | 78.97          | 1.511       | 190.1         | 34.32          | .9902       | 166.1         |     |
| $\infty^*$ | 248.0          | 4.000       | $\infty$      | 130.8          | 2.210       | $\infty$      | 78.92          | 1.509       | $\infty$      | 34.29          | .9897       | $\infty$      |     |
|            | $k_X = -4$     |             |               | $k_X = -2$     |             |               | $k_X = -1$     |             |               | $k_X = 0$      |             |               |     |
| 1.00       | 1 089          | 73.71       | 5.528         | 594.8          | 38.09       | 5.047         | 364.1          | 22.31       | 4.656         | 152.6          | 8.771       | 3.978         | 1.0 |
| 1.25       | 835.5          | 44.32       | 5.326         | 451.4          | 22.74       | 4.834         | 273.1          | 13.25       | 4.442         | 111.1          | 5.161       | 3.788         |     |
| 1.50       | 684.6          | 30.22       | 5.281         | 366.7          | 15.38       | 4.796         | 219.8          | 8.920       | 4.419         | 87.02          | 3.469       | 3.801         |     |
| 1.75       | 586.3          | 22.57       | 5.343         | 312.2          | 11.39       | 4.884         | 185.6          | 6.582       | 4.526         | 71.86          | 2.589       | 3.932         |     |
| 2.00       | 519.7          | 17.72       | 5.512         | 275.0          | 8.950       | 5.066         | 162.4          | 5.215       | 4.711         | 61.70          | 2.100       | 4.102         |     |
| 2.50       | 435.3          | 12.51       | 6.015         | 228.6          | 6.333       | 5.594         | 133.7          | 3.674       | 5.272         | 49.33          | 1.537       | 4.637         |     |
| 3.00       | 385.9          | 9.841       | 6.667         | 201.6          | 4.993       | 6.261         | 117.2          | 2.923       | 5.932         | 42.32          | 1.268       | 5.241         |     |
| 4.00       | 332.5          | 7.293       | 8.195         | 172.8          | 3.736       | 7.763         | 99.53          | 2.207       | 7.443         | 35.02          | 1.018       | 6.591         |     |
| 5.00       | 305.5          | 6.134       | 9.867         | 158.4          | 3.165       | 9.404         | 90.97          | 1.910       | 8.938         | 31.46          | .9106       | 7.996         |     |
| 7.00       | 279.8          | 5.123       | 13.39         | 144.8          | 2.672       | 12.81         | 82.85          | 1.636       | 12.23         | 27.46          | .7955       | 11.29         |     |
| 10.00      | 264.7          | 4.570       | 18.85         | 136.9          | 2.405       | 18.08         | 78.17          | 1.490       | 17.31         | 26.34          | .7698       | 15.59         |     |
| 15.00      | 255.9          | 4.263       | 28.10         | 132.3          | 2.258       | 27.04         | 75.48          | 1.409       | 25.98         | 25.28          | .7439       | 23.35         |     |
| 20.00      | 252.6          | 4.151       | 37.45         | 130.6          | 2.205       | 36.05         | 74.49          | 1.381       | 34.60         | 24.90          | .7344       | 31.17         |     |
| 50.00*     | 248.7          | 4.019       | 94.31         | 128.6          | 2.143       | 90.78         | 73.30          | 1.347       | 87.27         | 24.44          | .7240       | 83.56         |     |
| 100.00*    | 248.2          | 4.005       | 188.5         | 128.3          | 2.136       | 181.5         | 73.17          | 1.344       | 174.5         | 24.39          | .7228       | 167.1         |     |
| $\infty^*$ | 248.0          | 4.000       | $\infty$      | 128.3          | 2.134       | $\infty$      | 73.16          | 1.343       | $\infty$      | 24.35          | .7223       | $\infty$      |     |
|            | $k_X = -4$     |             |               | $k_X = -2$     |             |               | $k_X = -1$     |             |               | $k_X = 0$      |             |               |     |
| 1.00       | 1 231          | 98.00       | 6.933         | 635.3          | 47.94       | 6.439         | 354.7          | 25.55       | 5.994         | 97.89          | 6.396       | 4.978         | 2.0 |
| 1.25       | 951.0          | 59.19       | 6.694         | 487.6          | 28.71       | 6.181         | 270.3          | 15.25       | 5.718         | 72.87          | 3.799       | 4.670         |     |
| 1.50       | 780.4          | 40.10       | 6.595         | 398.3          | 19.41       | 6.066         | 219.6          | 10.26       | 5.591         | 58.14          | 2.617       | 4.539         |     |
| 1.75       | 667.9          | 29.52       | 6.591         | 339.7          | 14.27       | 6.051         | 186.6          | 7.537       | 5.568         | 48.59          | 1.949       | 4.533         |     |
| 2.00       | 589.3          | 23.04       | 6.657         | 299.0          | 11.14       | 6.108         | 163.7          | 5.896       | 5.626         | 41.99          | 1.542       | 4.624         |     |
| 2.50       | 488.6          | 15.89       | 6.935         | 247.2          | 7.675       | 6.388         | 134.7          | 4.096       | 5.921         | 33.98          | 1.142       | 4.976         |     |
| 3.00       | 428.3          | 12.20       | 7.357         | 216.4          | 5.936       | 6.824         | 117.7          | 3.180       | 6.385         | 29.28          | .9353       | 5.461         |     |
| 4.00       | 361.7          | 8.664       | 8.488         | 182.6          | 4.250       | 8.005         | 99.12          | 2.318       | 7.593         | 24.30          | .7395       | 6.682         |     |
| 5.00       | 327.0          | 7.051       | 9.853         | 165.3          | 3.494       | 9.388         | 89.65          | 1.931       | 8.984         | 21.81          | .6542       | 8.019         |     |
| 7.00       | 293.1          | 5.631       | 12.91         | 148.5          | 2.831       | 12.44         | 80.57          | 1.595       | 11.97         | 19.48          | .5818       | 10.85         |     |
| 10.00      | 272.5          | 4.847       | 17.82         | 138.3          | 2.468       | 17.29         | 75.18          | 1.411       | 16.75         | 18.12          | .5434       | 15.26         |     |
| 15.00      | 260.0          | 4.401       | 26.28         | 132.3          | 2.264       | 25.60         | 71.99          | 1.309       | 24.86         | 17.34          | .5223       | 22.76         |     |
| 20.00      | 255.1          | 4.235       | 34.87         | 129.9          | 2.187       | 34.10         | 70.78          | 1.272       | 33.08         | 17.05          | .5148       | 30.32         |     |
| 50.00*     | 249.0          | 4.032       | 87.35         | 127.1          | 2.097       | 85.34         | 69.29          | 1.227       | 83.18         | 16.69          | .5059       | 76.54         |     |
| 100.00*    | 248.3          | 4.008       | 174.6         | 126.8          | 2.087       | 170.6         | 69.11          | 1.222       | 166.3         | 16.65          | .5050       | 153.0         |     |
| $\infty^*$ | 248.0          | 4.000       | $\infty$      | 126.7          | 2.085       | $\infty$      | 69.05          | 1.221       | $\infty$      | 16.65          | .5047       | $\infty$      |     |

\*Denotes preflutter solution for this value of a/b.

TABLE II.- FLUTTER SOLUTIONS FOR PANELS WITH LENGTH-WIDTH RATIOS GREATER THAN OR EQUAL TO ONE - Continued

| a/b        | $\lambda_{cr}$ | $\phi_{cr}$ | $\alpha_{cr}$ | $\lambda_{cr}$ | $\phi_{cr}$ | $\alpha_{cr}$ | $\lambda_{cr}$ | $\phi_{cr}$ | $\alpha_{cr}$ | $\lambda_{cr}$ | $\phi_{cr}$ | $\alpha_{cr}$ | r    |
|------------|----------------|-------------|---------------|----------------|-------------|---------------|----------------|-------------|---------------|----------------|-------------|---------------|------|
|            | $k_X = 1$      |             |               | $k_X = 2$      |             |               | $k_X = 3$      |             |               | $k_X = 4$      |             |               |      |
| 1.00       | 426.0          | 15.37       | 2.747         | 343.4          | 11.79       | 2.477         | 264.6          | 8.200       | 2.190         | 190.9          | 5.287       | 1.818         | 0.0  |
| 1.25       | 242.8          | 7.692       | 2.897         | 175.8          | 5.419       | 2.477         | 114.0          | 3.325       | 1.981         | 58.13          | 1.425       | 1.324         |      |
| 1.50       | 158.5          | 4.757       | 3.059         | 101.7          | 3.131       | 2.477         | 51.31          | 1.690       | 1.724         | 8.359          | .4618       | .4498         |      |
| 1.75       | 113.6          | 3.367       | 3.246         | 64.07          | 2.151       | 2.477         | 21.95          | 1.121       | 1.364         | **             |             |               |      |
| 2.00       | 87.14          | 2.620       | 3.456         | 42.92          | 1.674       | 2.477         | 7.247          | .9106       | .8275         |                |             |               |      |
| 2.50       | 58.80          | 1.877       | 3.976         | 21.97          | 1.276       | 2.477         | **             |             |               |                |             |               |      |
| 3.00       | 45.07          | 1.600       | 4.405         | 12.72          | 1.133       | 2.477         |                |             |               |                |             |               |      |
| 4.00       | 32.29          | 1.342       | 5.477         | 5.365          | 1.042       | 2.477         |                |             |               |                |             |               |      |
| 5.00       | 26.66          | 1.243       | 6.539         | 2.747          | 1.017       | 2.477         |                |             |               |                |             |               |      |
| 7.00       | 21.93          | 1.160       | 9.002         | 1.001          | 1.005       | 2.477         |                |             |               |                |             |               |      |
| 10.00      | 19.41          | 1.120       | 12.70         | .3434          | 1.001       | 2.477         |                |             |               |                |             |               |      |
| 15.00      | 18.03          | 1.099       | 18.97         | .1017          | 1.000       | 2.477         |                |             |               |                |             |               |      |
| 20.00      | 17.54          | 1.092       | 25.32         | .0429          | 1.000       | 2.477         |                |             |               |                |             |               |      |
| 50.00*     | 16.96          | 1.084       | 64.18         | .0027          | 1.000       | 2.565         |                |             |               |                |             |               |      |
| 100.00*    | 16.90          | 1.084       | 128.3         | .0003          | 1.000       | 2.565         |                |             |               |                |             |               |      |
| $\infty^*$ | 16.88          | 1.083       | $\infty$      | .0000          | 1.000       | 2.565         |                |             |               |                |             |               |      |
|            | $k_X = 0.9$    |             |               | $k_X = 1.859$  |             |               | $k_X = 2.7$    |             |               | $k_X = 3.6$    |             |               |      |
| 1.00       | 354.2          | 13.21       | 2.698         | 265.6          | 9.506       | 2.414         | 193.2          | 6.514       | 2.108         | 122.0          | 3.444       | 1.705         | 0.05 |
| 1.25       | 210.8          | 6.926       | 2.859         | 141.2          | 4.588       | 2.424         | 86.11          | 2.727       | 1.930         | 34.36          | .9302       | 1.159         |      |
| 1.50       | 141.3          | 4.328       | 3.058         | 83.48          | 2.726       | 2.439         | 39.39          | 1.483       | 1.683         | .8032          | .3605       | .06165        |      |
| 1.75       | 103.2          | 3.145       | 3.242         | 53.27          | 1.919       | 2.448         | 16.90          | 1.035       | 1.330         | **             |             |               |      |
| 2.00       | 80.30          | 2.463       | 3.475         | 36.00          | 1.520       | 2.458         | 5.447          | .8706       | .7860         |                |             |               |      |
| 2.50       | 55.41          | 1.819       | 3.953         | 18.63          | 1.189       | 2.458         | **             |             |               |                |             |               |      |
| 3.00       | 42.93          | 1.530       | 4.473         | 10.84          | 1.067       | 2.459         |                |             |               |                |             |               |      |
| 4.00       | 31.27          | 1.288       | 5.590         | 4.599          | .9883       | 2.479         |                |             |               |                |             |               |      |
| 5.00       | 26.12          | 1.192       | 6.749         | 2.361          | .9672       | 2.479         |                |             |               |                |             |               |      |
| 7.00       | 21.69          | 1.113       | 9.242         | .8626          | .9563       | 2.479         |                |             |               |                |             |               |      |
| 10.00      | 19.33          | 1.076       | 13.00         | .2962          | .9533       | 2.480         |                |             |               |                |             |               |      |
| 15.00      | 18.04          | 1.056       | 19.44         | .0878          | .9525       | 2.480         |                |             |               |                |             |               |      |
| 20.00      | 17.57          | 1.049       | 25.96         | .0371          | .9525       | 2.481         |                |             |               |                |             |               |      |
| 50.00*     | 17.03          | 1.041       | 65.78         | .0023          | .9524       | 2.565         |                |             |               |                |             |               |      |
| 100.00*    | 16.98          | 1.040       | 131.5         | .0003          | .9524       | 2.565         |                |             |               |                |             |               |      |
| $\infty^*$ | 16.96          | 1.040       | $\infty$      | .0000          | .9524       | 2.565         |                |             |               |                |             |               |      |
|            | $k_X = 0.7$    |             |               | $k_X = 1.4$    |             |               | $k_X = 1.528$  |             |               | $k_X = 2.1$    |             |               |      |
| 1.00       | 237.8          | 10.20       | 2.692         | 157.6          | 6.537       | 2.427         | 143.9          | 5.883       | 2.374         | 86.24          | 3.376       | 2.024         | 0.2  |
| 1.25       | 152.1          | 5.542       | 2.840         | 91.48          | 3.371       | 2.468         | 81.26          | 3.003       | 2.384         | 39.50          | 1.578       | 1.831         |      |
| 1.50       | 107.2          | 3.637       | 3.017         | 58.08          | 2.177       | 2.507         | 49.97          | 1.940       | 2.382         | 17.69          | 1.010       | 1.540         |      |
| 1.75       | 81.21          | 2.673       | 3.233         | 39.48          | 1.606       | 2.578         | 32.74          | 1.445       | 2.399         | 6.807          | .8136       | 1.092         |      |
| 2.00       | 64.97          | 2.139       | 3.467         | 28.32          | 1.318       | 2.647         | 22.53          | 1.198       | 2.412         | 1.209          | .7419       | .3593         |      |
| 2.50       | 46.66          | 1.597       | 4.004         | 16.47          | 1.060       | 2.800         | 11.93          | .9876       | 2.422         | **             |             |               |      |
| 3.00       | 37.15          | 1.358       | 4.547         | 10.81          | .9635       | 2.921         | 7.034          | .9079       | 2.443         |                |             |               |      |
| 4.00       | 27.99          | 1.150       | 5.704         | 5.906          | .8879       | 3.305         | 3.025          | .8574       | 2.456         |                |             |               |      |
| 5.00       | 23.83          | 1.063       | 6.959         | 3.940          | .8634       | 3.718         | 1.563          | .8432       | 2.461         |                |             |               |      |
| 7.00       | 20.21          | .9938       | 9.536         | 2.419          | .8476       | 4.636         | .5743          | .8359       | 2.479         |                |             |               |      |
| 10.00      | 18.24          | .9586       | 13.53         | 1.668          | .8411       | 5.883         | .1978          | .8340       | 2.479         |                |             |               |      |
| 15.00      | 17.16          | .9404       | 20.28         | 1.314          | .8377       | 8.879         | .05875         | .8334       | 2.479         |                |             |               |      |
| 20.00      | 16.77          | .9344       | 27.07         | 1.183          | .8369       | 11.72         | .02480         | .8333       | 2.479         |                |             |               |      |
| 50.00*     | 16.31          | .9270       | 68.48         | 1.032          | .8358       | 29.66         | .0016          | .8333       | 2.565         |                |             |               |      |
| 100.00*    | 16.26          | .9263       | 136.9         | 1.014          | .8357       | 59.16         | .0002          | .8333       | 2.565         |                |             |               |      |
| $\infty^*$ | 16.24          | .9260       | $\infty$      | 1.009          | .8357       | $\infty$      | .0000          | .8333       | 2.565         |                |             |               |      |

\*Denotes flutter solution for this value of a/b.

\*\*A &gt; 5, see appendix C.

TABLE II.- FLUTTER SOLUTIONS FOR PANELS WITH LENGTH-WIDTH RATIOS GREATER THAN OR EQUAL TO ONE - Concluded

| a/b        | $\lambda_{cr}$ | $\phi_{cr}$ | $\alpha_{cr}$ | $\lambda_{cr}$ | $\phi_{cr}$ | $\alpha_{cr}$ | $\lambda_{cr}$ | $\phi_{cr}$ | $\alpha_{cr}$ | $\lambda_{cr}$ | $\phi_{cr}$ | $\alpha_{cr}$ | r   |
|------------|----------------|-------------|---------------|----------------|-------------|---------------|----------------|-------------|---------------|----------------|-------------|---------------|-----|
|            | $k_x = 0.5$    |             |               | $k_x = 1.0$    |             |               | $k_x = 1.224$  |             |               | $k_x = 1.5$    |             |               |     |
| 1.00       | 173.2          | 8.178       | 2.917         | 105.2          | 4.904       | 2.620         | 77.25          | 3.602       | 2.437         | 45.95          | 2.172       | 2.132         | 0.4 |
| 1.25       | 116.5          | 4.661       | 2.967         | 65.69          | 2.725       | 2.621         | 45.29          | 1.989       | 2.384         | 23.02          | 1.219       | 1.928         |     |
| 1.50       | 85.50          | 3.127       | 3.112         | 44.65          | 1.844       | 2.684         | 28.60          | 1.373       | 2.361         | 11.63          | .8744       | 1.717         |     |
| 1.75       | 66.87          | 2.320       | 3.336         | 32.36          | 1.418       | 2.781         | 19.11          | 1.083       | 2.368         | 5.578          | .7507       | 1.412         |     |
| 2.00       | 54.89          | 1.883       | 3.564         | 24.68          | 1.186       | 2.901         | 13.34          | .9362       | 2.379         | 2.240          | .6975       | .9850         |     |
| 2.50       | 40.49          | 1.382       | 4.212         | 16.11          | .9615       | 3.205         | 7.201          | .8072       | 2.413         | **             |             |               |     |
| 3.00       | 33.48          | 1.216       | 4.674         | 11.76          | .8690       | 3.511         | 4.297          | .7602       | 2.424         |                |             |               |     |
| 4.00       | 26.08          | 1.027       | 5.922         | 7.706          | .7938       | 4.212         | 1.871          | .7289       | 2.458         |                |             |               |     |
| 5.00       | 22.64          | .9475       | 7.227         | 5.943          | .7650       | 4.999         | .9731          | .7205       | 2.458         |                |             |               |     |
| 7.00       | 19.59          | .8830       | 9.928         | 4.464          | .7439       | 6.623         | .3595          | .7160       | 2.459         |                |             |               |     |
| 10.00      | 17.91          | .8500       | 14.09         | 3.688          | .7341       | 9.191         | .1242          | .7147       | 2.479         |                |             |               |     |
| 15.00      | 16.98          | .8326       | 21.12         | 3.267          | .7291       | 13.67         | .0370          | .7144       | 2.479         |                |             |               |     |
| 20.00      | 16.64          | .8262       | 28.29         | 3.116          | .7275       | 18.19         | .0156          | .7143       | 2.480         |                |             |               |     |
| 50.00*     | 16.24          | .8197       | 71.48         | 2.937          | .7255       | 46.22         | .0010          | .7143       | 2.565         |                |             |               |     |
| 100.00*    | 16.20          | .8190       | 142.9         | 2.918          | .7253       | 92.34         | .0001          | .7143       | 2.565         |                |             |               |     |
| $\infty^*$ | 16.19          | .8187       | $\infty$      | 2.912          | .7253       | $\infty$      | .0000          | .7143       | 2.565         |                |             |               |     |
|            | $k_x = 0.25$   |             |               | $k_x = 0.50$   |             |               | $k_x = 0.75$   |             |               | $k_x = 0.80$   |             |               |     |
| 1.00       | 104.8          | 5.948       | 3.697         | 60.29          | 3.417       | 3.307         | 21.27          | 1.342       | 2.657         | 14.66          | 1.046       | 2.423         | 1.0 |
| 1.25       | 74.85          | 3.500       | 3.520         | 41.53          | 2.040       | 3.149         | 13.04          | .8954       | 2.466         | 8.330          | .7238       | 2.200         |     |
| 1.50       | 57.65          | 2.390       | 3.535         | 30.89          | 1.434       | 3.162         | 8.507          | .7044       | 2.399         | 4.947          | .5999       | 2.055         |     |
| 1.75       | 46.89          | 1.799       | 3.671         | 24.32          | 1.125       | 3.268         | 5.830          | .6172       | 2.368         | 3.004          | .5465       | 1.918         |     |
| 2.00       | 39.73          | 1.467       | 3.856         | 20.00          | .9488       | 3.427         | 4.153          | .5709       | 2.371         | 1.831          | .5217       | 1.756         |     |
| 2.50       | 31.07          | 1.117       | 4.335         | 14.88          | .7690       | 3.843         | 2.305          | .5307       | 2.379         | .6277          | .5020       | 1.338         |     |
| 3.00       | 26.23          | .9464       | 4.903         | 12.07          | .6862       | 4.308         | 1.400          | .5151       | 2.410         | .1194          | .4970       | .5663         |     |
| 4.00       | 21.24          | .7910       | 6.171         | 9.247          | .6129       | 5.366         | .6225          | .5051       | 2.424         | **             |             |               |     |
| 5.00       | 18.83          | .7258       | 7.465         | 7.918          | .5822       | 6.503         | .3270          | .5021       | 2.449         |                |             |               |     |
| 7.00       | 16.64          | .6697       | 10.28         | 6.729          | .5577       | 8.887         | .1220          | .5006       | 2.456         |                |             |               |     |
| 10.00      | 15.41          | .6408       | 14.57         | 6.069          | .5451       | 12.56         | .0424          | .5001       | 2.466         |                |             |               |     |
| 15.00      | 14.71          | .6254       | 21.82         | 5.700          | .5385       | 18.76         | .0126          | .5000       | 2.479         |                |             |               |     |
| 20.00      | 14.45          | .6199       | 29.16         | 5.565          | .5362       | 25.00         | .0053          | .5000       | 2.479         |                |             |               |     |
| 50.00*     | 14.15          | .6136       | 73.74         | 5.405          | .5336       | 63.48         | .0003          | .5000       | 2.565         |                |             |               |     |
| 100.00*    | 14.11          | .6129       | 147.4         | 5.389          | .5333       | 126.9         | .00004         | .5000       | 2.565         |                |             |               |     |
| $\infty^*$ | 14.10          | .6127       | $\infty$      | 5.383          | .5332       | $\infty$      | .00000         | .5000       | 2.565         |                |             |               |     |
|            | $k_x = 0.125$  |             |               | $k_x = 0.25$   |             |               | $k_x = 0.375$  |             |               | $k_x = 0.4444$ |             |               |     |
| 1.00       | 69.31          | 4.517       | 4.709         | 42.09          | 2.742       | 4.312         | 17.30          | 1.219       | 3.628         | 5.545          | 0.5716      | 2.836         | 2.0 |
| 1.25       | 51.11          | 2.695       | 4.395         | 30.35          | 1.690       | 3.970         | 12.01          | .8024       | 3.336         | 3.460          | .4428       | 2.569         |     |
| 1.50       | 40.39          | 1.857       | 4.265         | 23.77          | 1.178       | 3.884         | 8.964          | .6157       | 3.249         | 2.297          | .3901       | 2.456         |     |
| 1.75       | 33.53          | 1.408       | 4.266         | 19.45          | .9231       | 3.891         | 6.996          | .5104       | 3.313         | 1.597          | .3661       | 2.399         |     |
| 2.00       | 28.86          | 1.142       | 4.361         | 16.57          | .7675       | 4.010         | 5.796          | .4672       | 3.374         | 1.151          | .3536       | 2.374         |     |
| 2.50       | 23.09          | .8584       | 4.729         | 13.01          | .6093       | 4.374         | 4.280          | .4122       | 3.697         | .6503          | .3420       | 2.384         |     |
| 3.00       | 19.77          | .7174       | 5.239         | 10.98          | .5319       | 4.859         | 3.441          | .3869       | 4.094         | .4000          | .3378       | 2.389         |     |
| 4.00       | 16.26          | .5902       | 6.414         | 8.871          | .4617       | 5.993         | 2.592          | .3651       | 5.006         | .1803          | .3348       | 2.407         |     |
| 5.00       | 14.52          | .5347       | 7.705         | 7.836          | .4320       | 7.207         | 2.190          | .3560       | 6.024         | .0955          | .3339       | 2.435         |     |
| 7.00       | 12.91          | .4873       | 10.47         | 6.882          | .4069       | 9.812         | 1.829          | .3488       | 8.107         | .0359          | .3335       | 2.445         |     |
| 10.00      | 11.97          | .4623       | 14.74         | 6.337          | .3938       | 13.84         | 1.628          | .3451       | 11.45         | .0125          | .3334       | 2.471         |     |
| 15.00      | 11.44          | .4488       | 22.00         | 6.026          | .3868       | 20.68         | 1.515          | .3431       | 17.14         | .0038          | .3334       | 2.477         |     |
| 20.00      | 11.24          | .4439       | 29.36         | 5.911          | .3843       | 27.60         | 1.474          | .3425       | 22.87         | .0016          | .3333       | 2.485         |     |
| 50.00*     | 11.00          | .4384       | 74.09         | 5.775          | .3814       | 69.78         | 1.424          | .3417       | 58.02         | .0001          | .3333       | 2.565         |     |
| 100.00*    | 10.97          | .4377       | 148.1         | 5.759          | .3811       | 139.5         | 1.420          | .3416       | 116.0         | .00001         | .3333       | 2.565         |     |
| $\infty^*$ | 10.96          | .4375       | $\infty$      | 5.753          | .3810       | $\infty$      | 1.418          | .3416       | $\infty$      | .00000         | .3333       | 2.565         |     |

\* Denotes preflutter solution for this value of a/b.

\*\* A &gt; 5, see appendix C.

TABLE III.- COMPARISON OF PREFLUTTER AND  
EXACT FLUTTER SOLUTIONS FOR  $\frac{a}{b} = 20$

| r         | k <sub>x</sub>                            | λ <sub>cr</sub>   | λ <sub>p</sub>  |
|-----------|---|---|---|
| 0<br>↓    | -4<br>-2<br>-1<br>0<br>1<br>2             | 249.7<br>136.4<br>88.87<br>48.68<br>17.54<br>.0429          | 249.3<br>136.0<br>88.58<br>48.45<br>17.39<br>.0422          |
| 0.05<br>↓ | -4<br>-2<br>-1<br>0<br>.9<br>1.859        | 249.8<br>135.6<br>87.31<br>46.37<br>17.57<br>.0371          | 249.4<br>135.3<br>87.02<br>46.15<br>17.43<br>.0364          |
| 0.2<br>↓  | -4<br>-2<br>-1<br>0<br>.7<br>1.4<br>1.528 | 250.2<br>133.9<br>83.59<br>40.64<br>16.77<br>1.183<br>.0248 | 249.8<br>133.6<br>83.31<br>40.46<br>16.65<br>1.153<br>.0244 |

| r        | k <sub>x</sub>                                       | λ <sub>cr</sub>  | λ <sub>p</sub>   |
|----------|--|--|--|
| 0.4<br>↓ | -4<br>-2<br>-1<br>0<br>.5<br>1.0<br>1.224            | 250.8<br>132.4<br>80.11<br>34.98<br>16.64<br>3.116<br>.0156          | 250.3<br>132.1<br>79.84<br>34.82<br>16.54<br>3.073<br>.0156          |
| 1<br>↓   | -4<br>-2<br>-1<br>0<br>.25<br>.5<br>.75              | 252.6<br>130.6<br>74.49<br>24.90<br>14.45<br>5.565<br>.0053          | 251.8<br>130.2<br>74.25<br>24.78<br>14.38<br>5.525<br>.0053          |
| 2<br>↓   | -4<br>-2<br>-1<br>0<br>.125<br>.250<br>.375<br>.4444 | 255.1<br>129.9<br>70.78<br>17.05<br>11.24<br>5.911<br>1.474<br>.0016 | 254.3<br>129.5<br>70.51<br>16.97<br>11.18<br>5.880<br>1.462<br>.0016 |

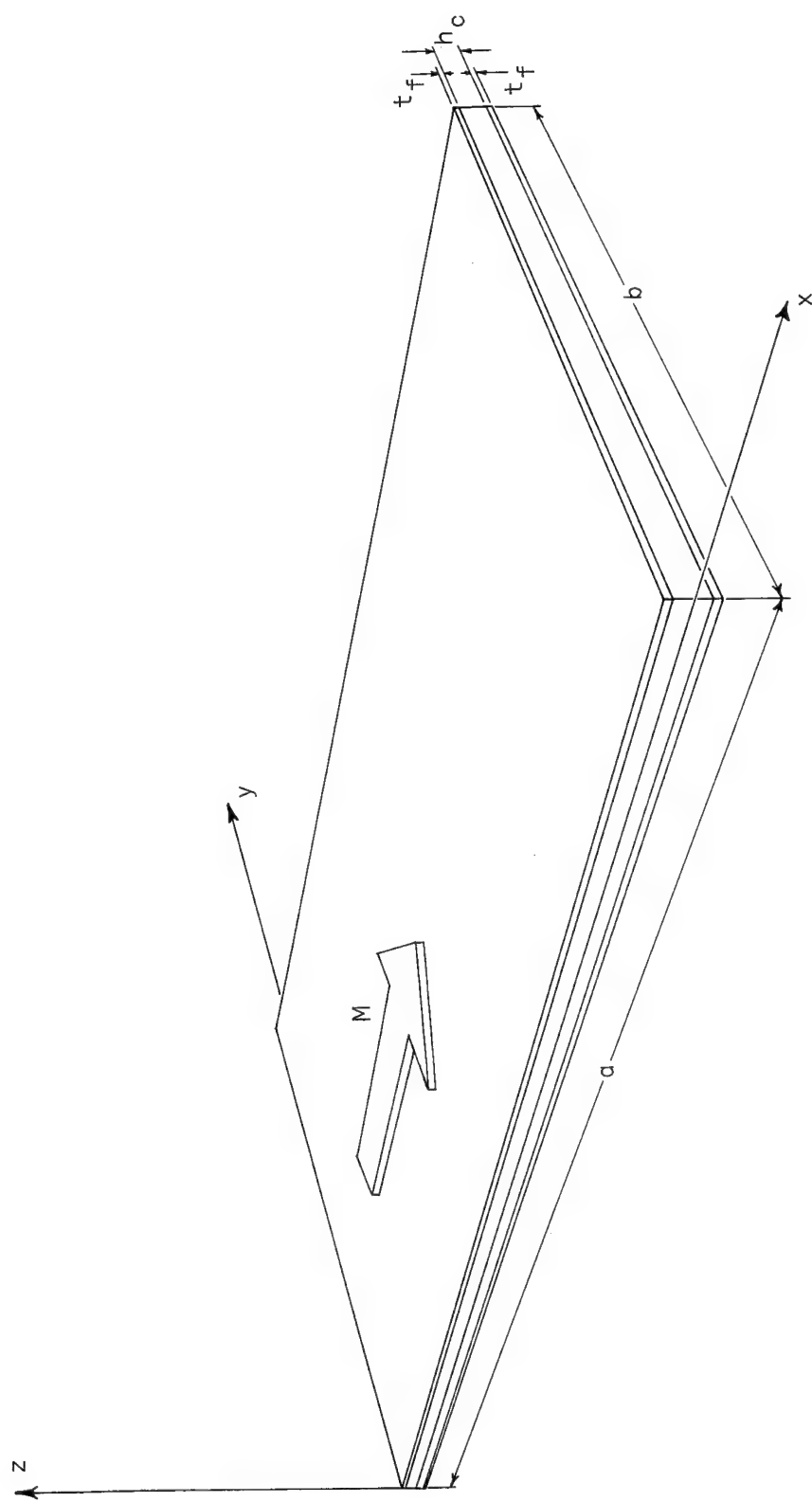


Figure 1.- Panel geometry, coordinate system, and flow direction.

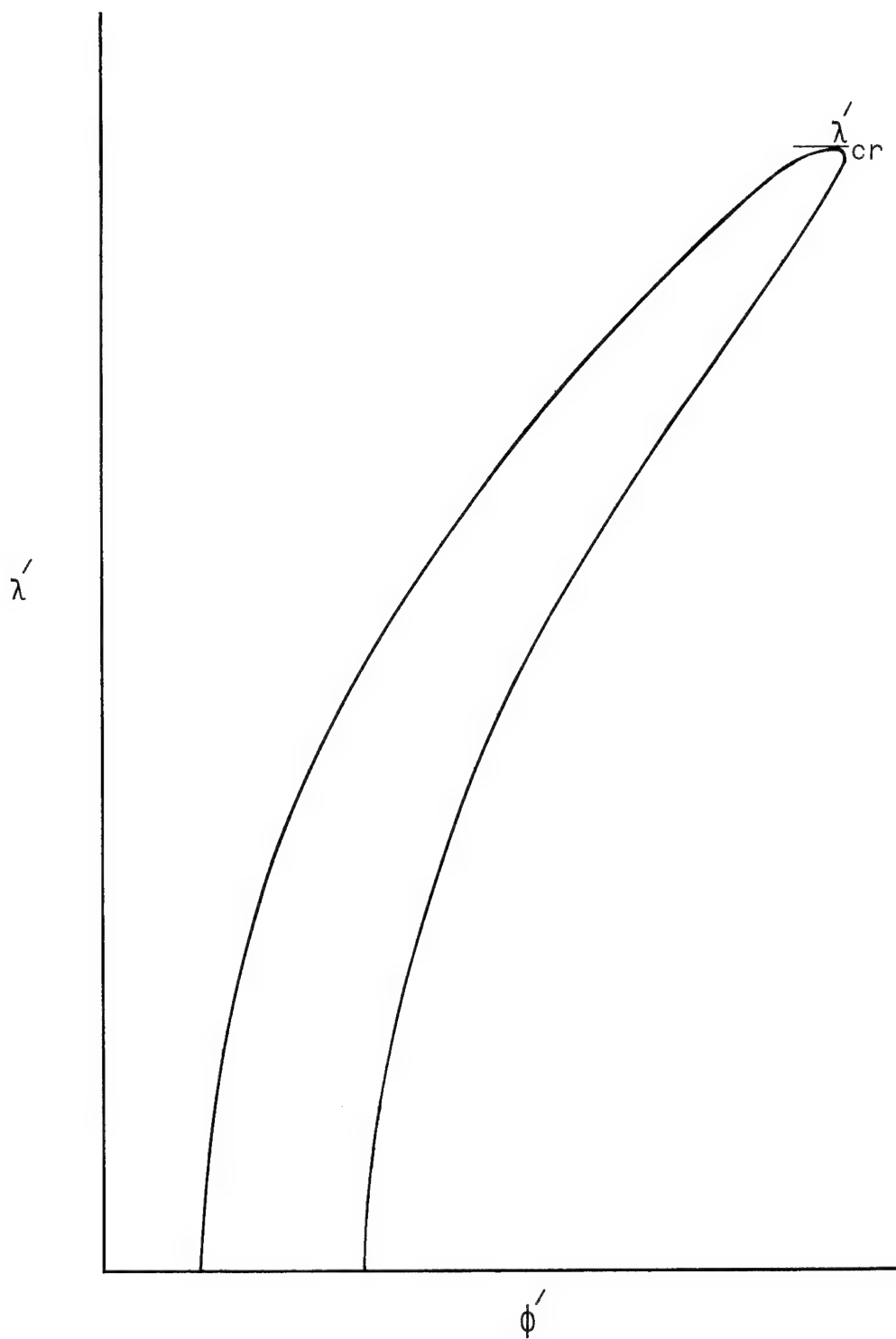
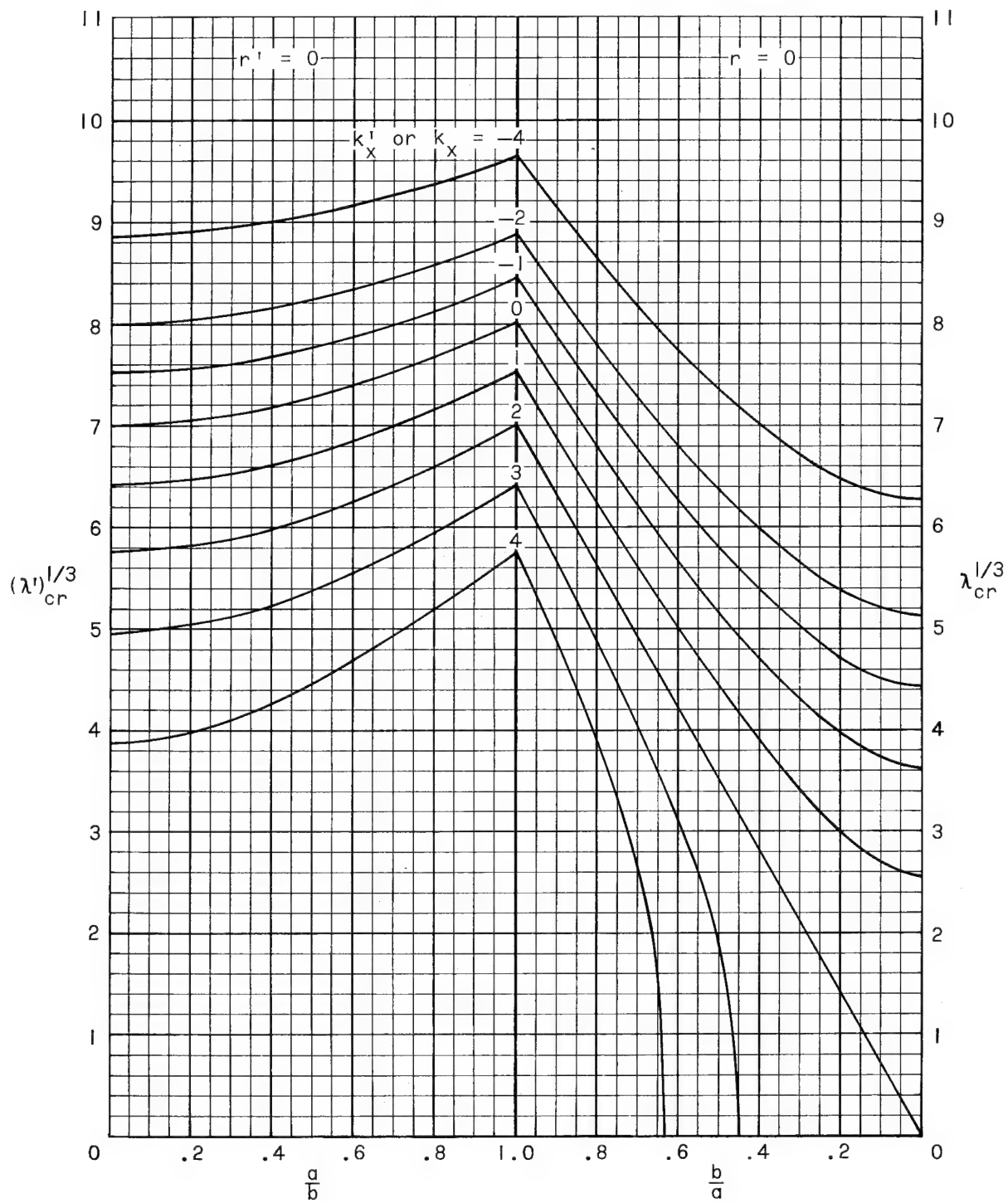


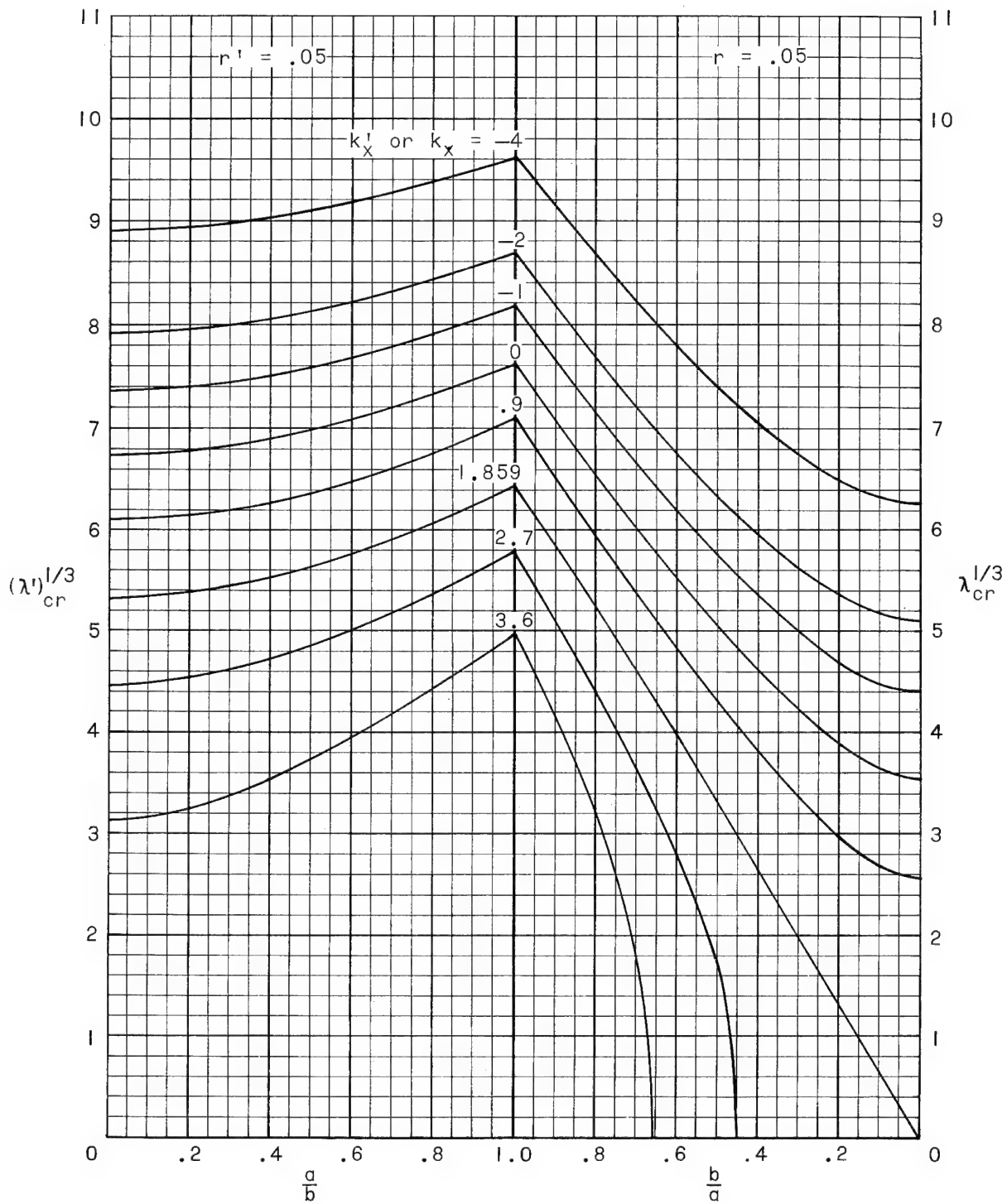
Figure 2.- Frequency loop illustrating  $\lambda'_{cr}$ .



(a)  $r$  or  $r' = 0$ .

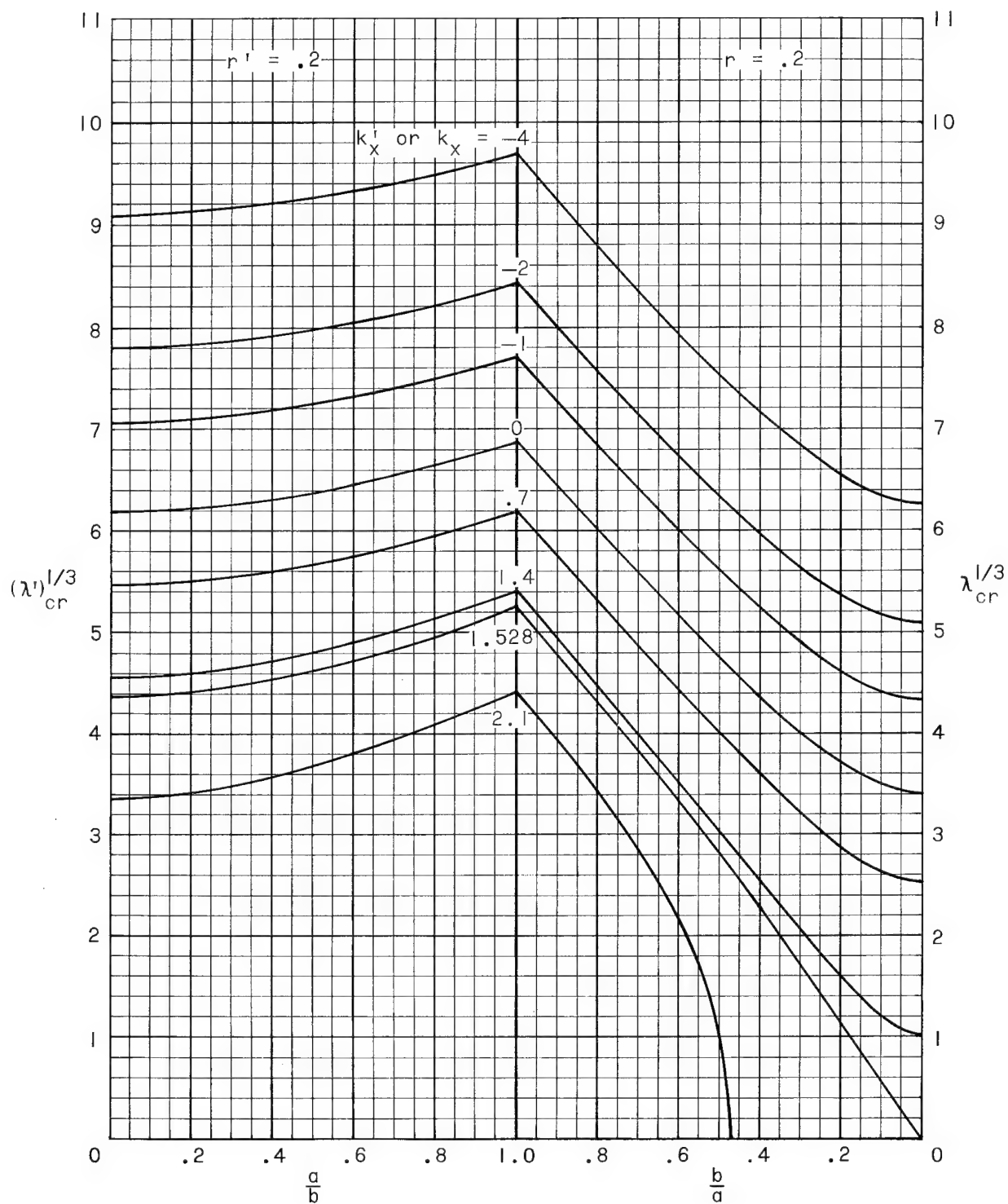
Figure 3.- Flutter values of dynamic pressure parameter.





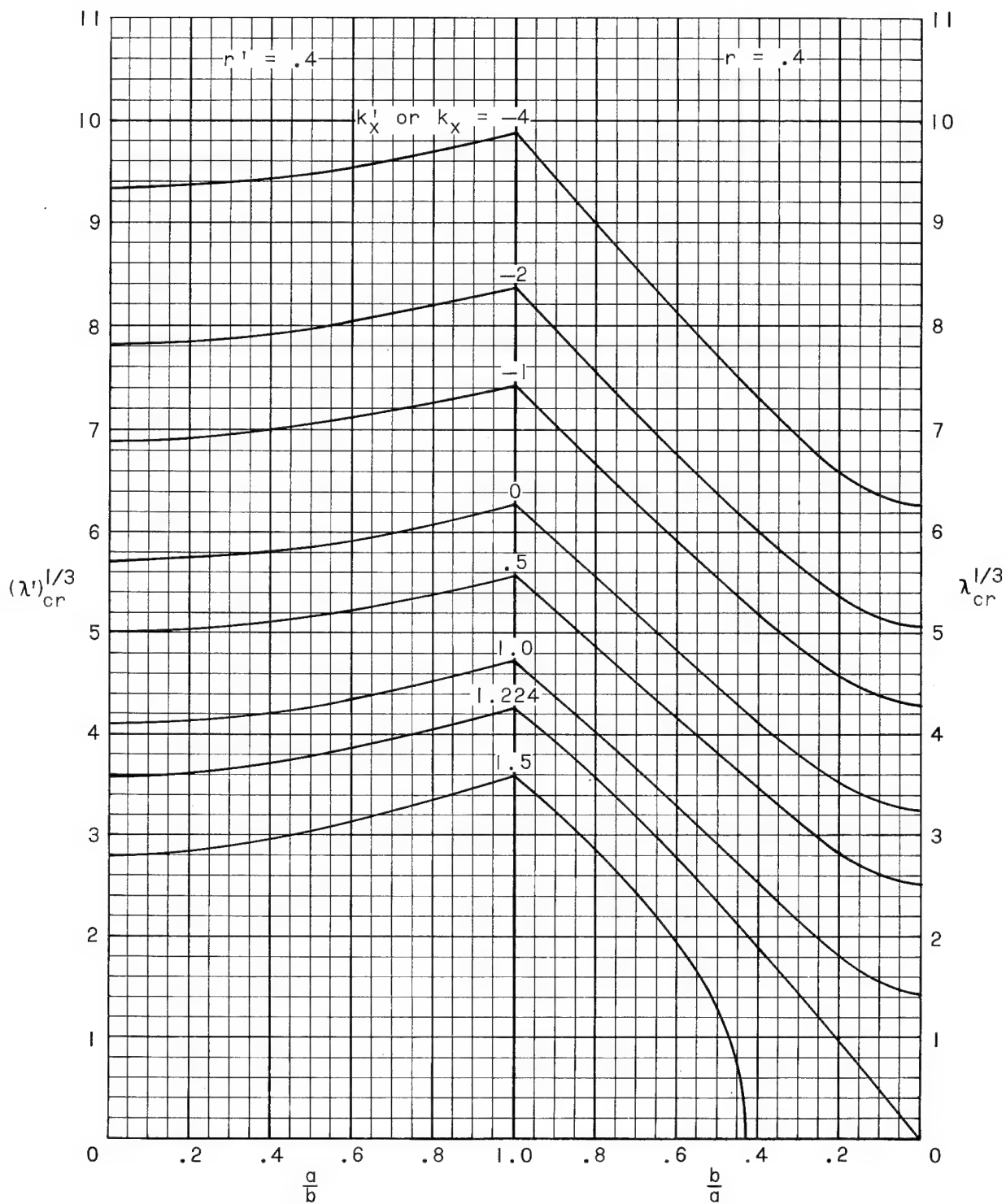
(b)  $r$  or  $r' = 0.05$ .

Figure 3.- Continued.



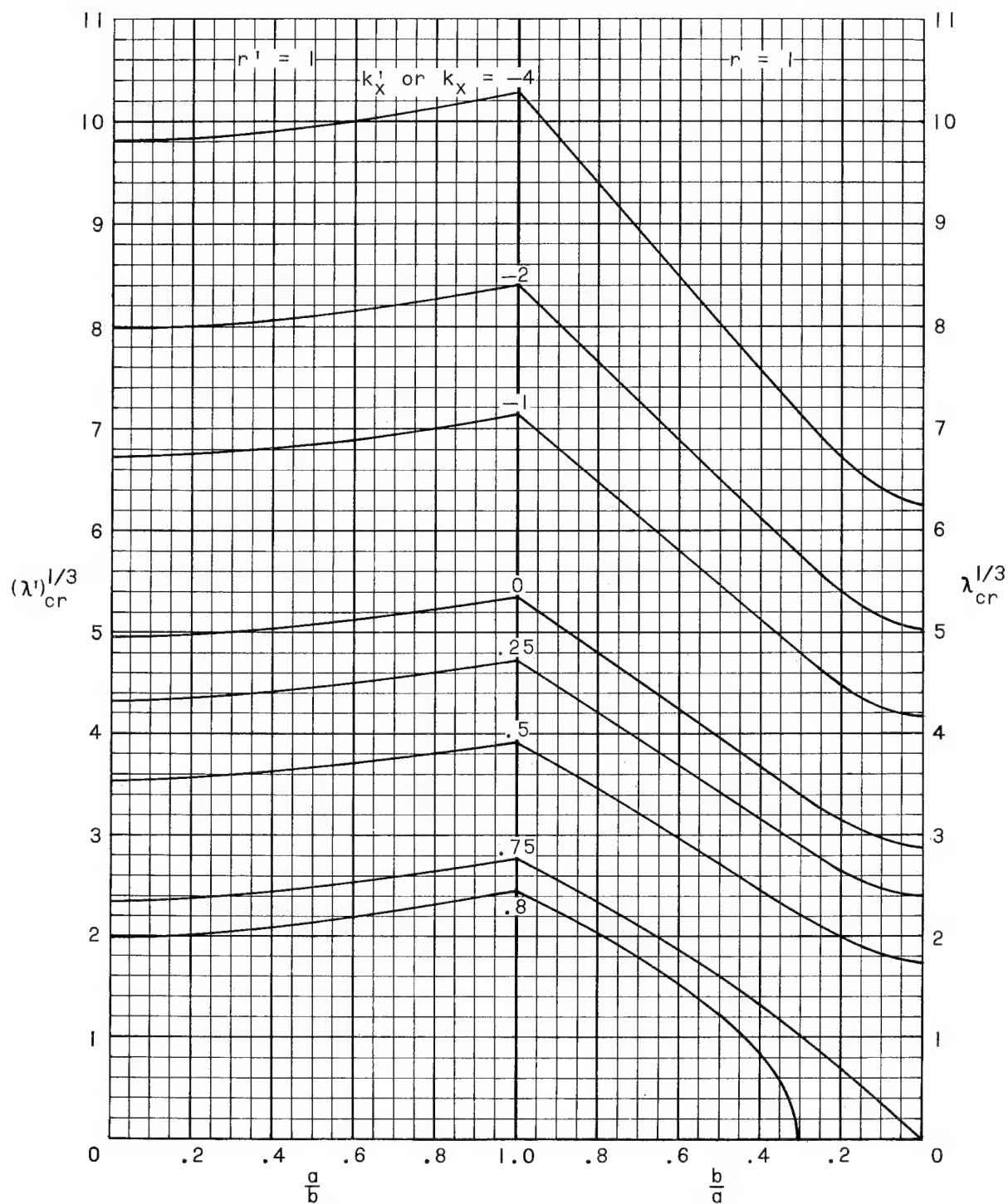
(c)  $r$  or  $r' = 0.2$ .

Figure 3.- Continued.



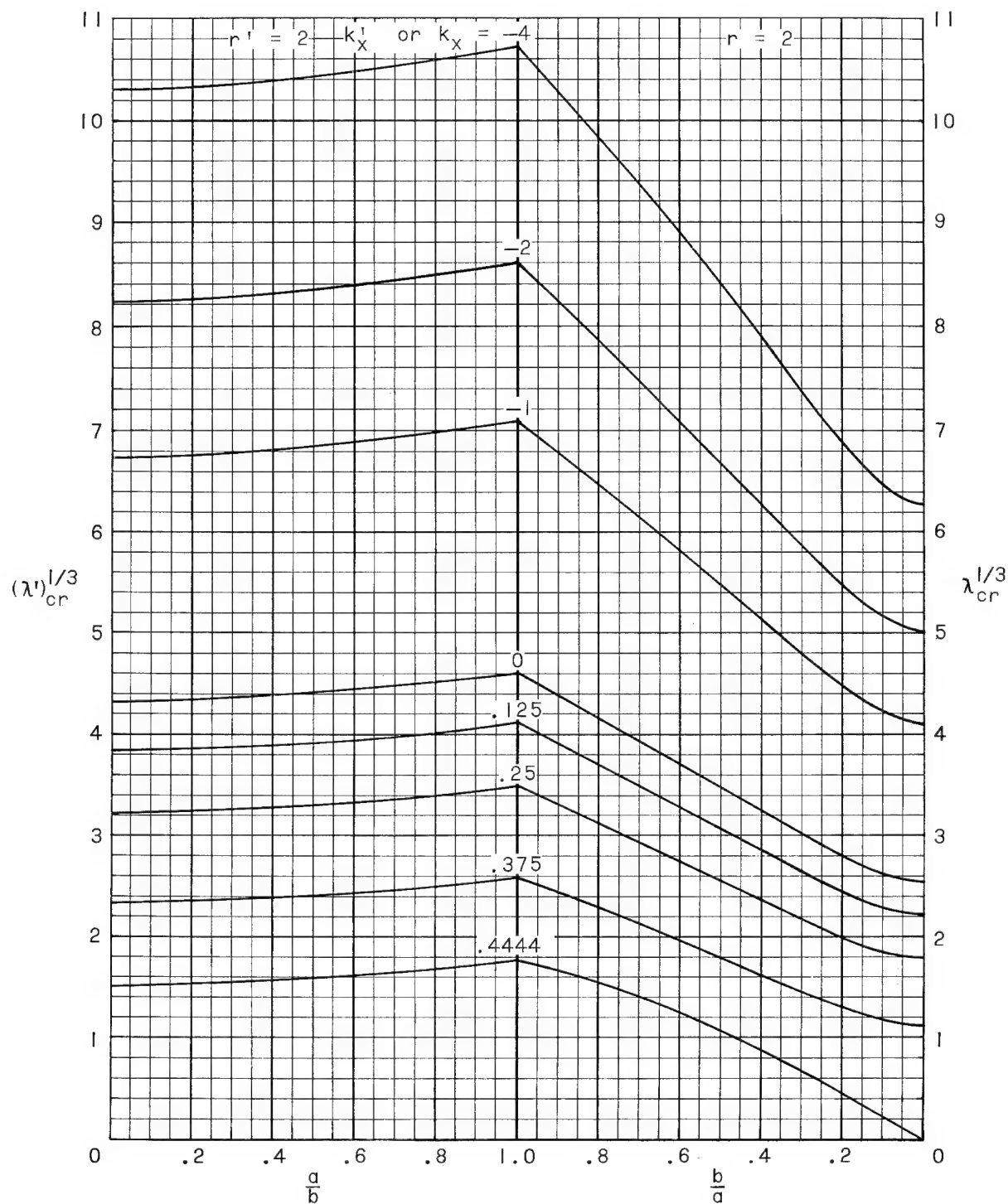
(d)  $r$  or  $r' = 0.4$ .

Figure 3.- Continued.



(e)  $r$  or  $r' = 1.0$ .

Figure 3.- Continued.



(f)  $r$  or  $r' = 2.0$ .

Figure 3.- Concluded.

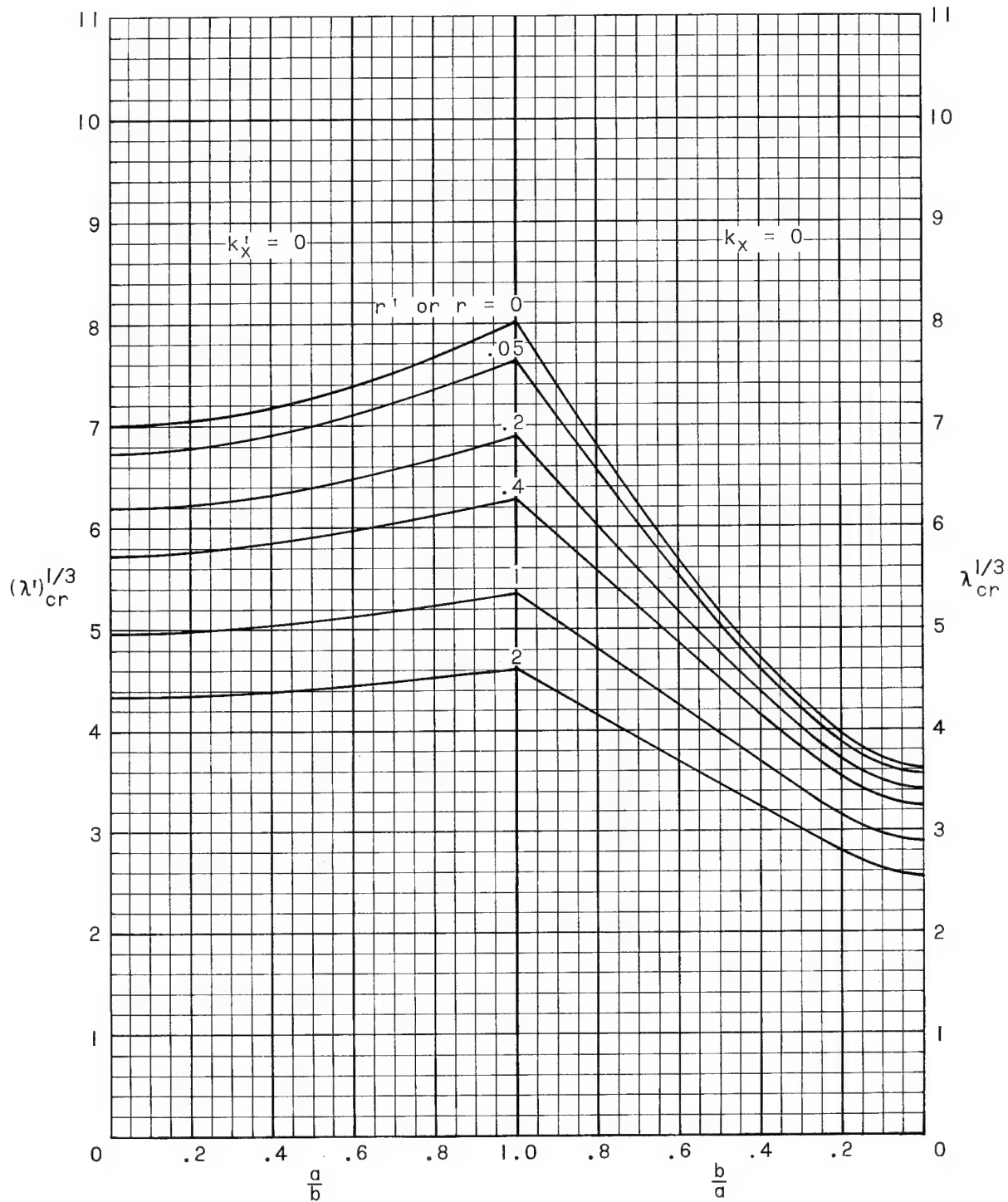


Figure 4.- Flutter values of dynamic-pressure parameter.  $k_x$  or  $k'_x = 0.0$ .

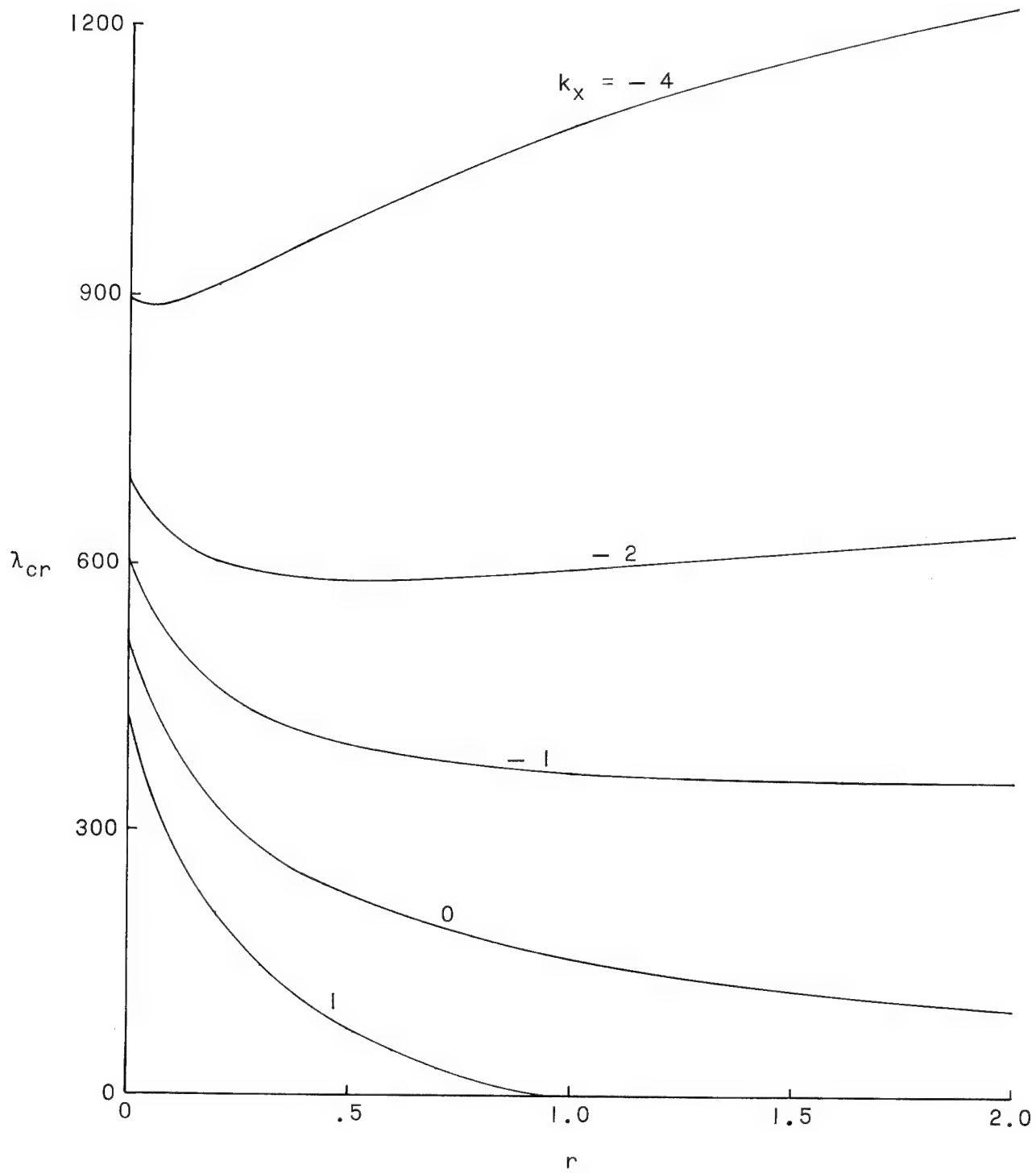


Figure 5.- Effect of transverse shear flexibility and stress on  $\lambda_{cr}$ .  $a/b = 1$ .

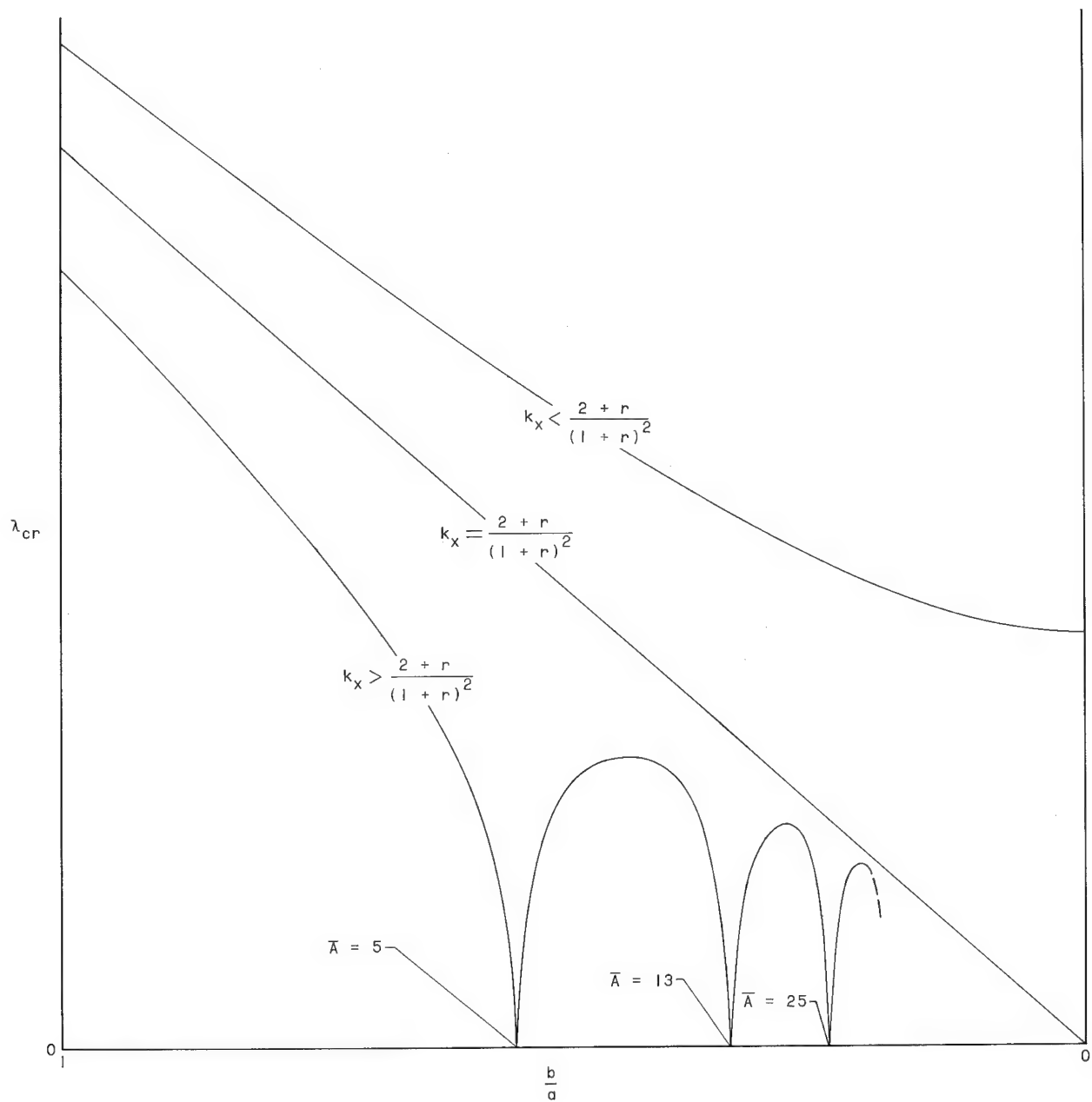


Figure 6.- Effect of stress on the flutter boundary for  $r$  constant.



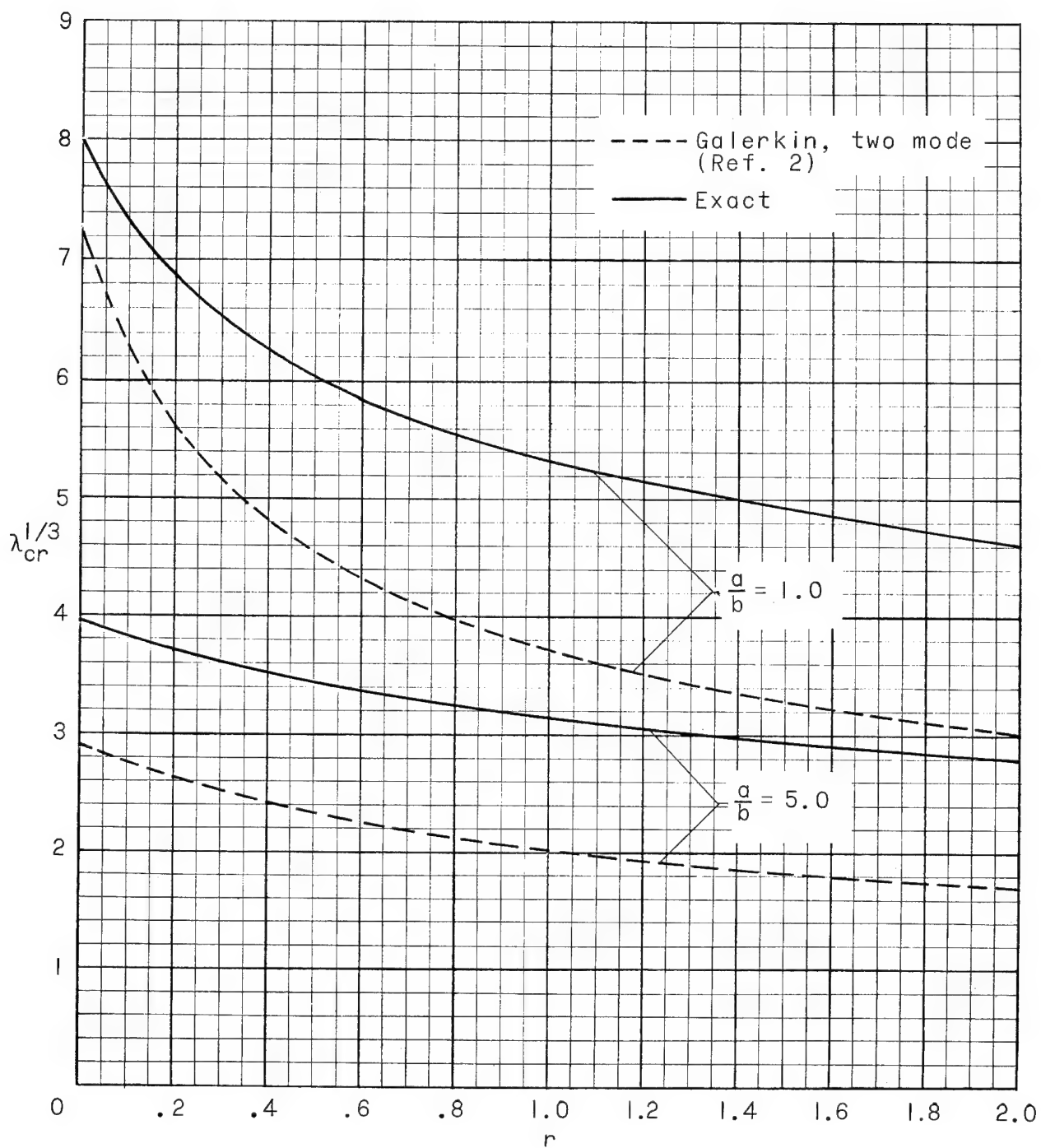


Figure 7.- Comparison of flutter boundaries from two-mode Galerkin solution and exact solution.  $k_x = 0$ .

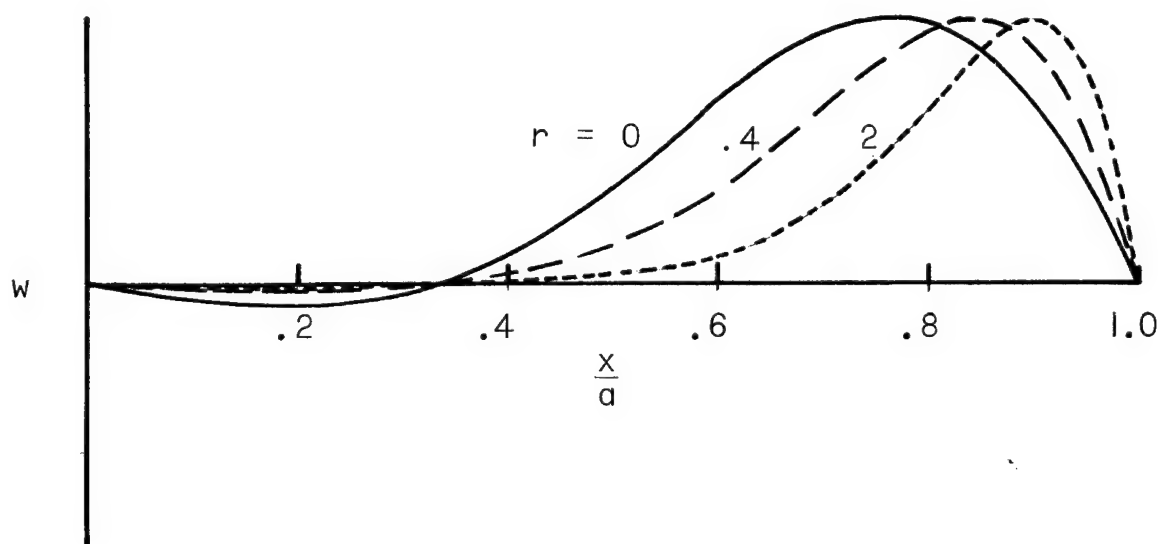


Figure 8.- Mode shape at flutter for an unstressed square sandwich panel.  $a/b = 1$ ;  $k_x = 0$ .

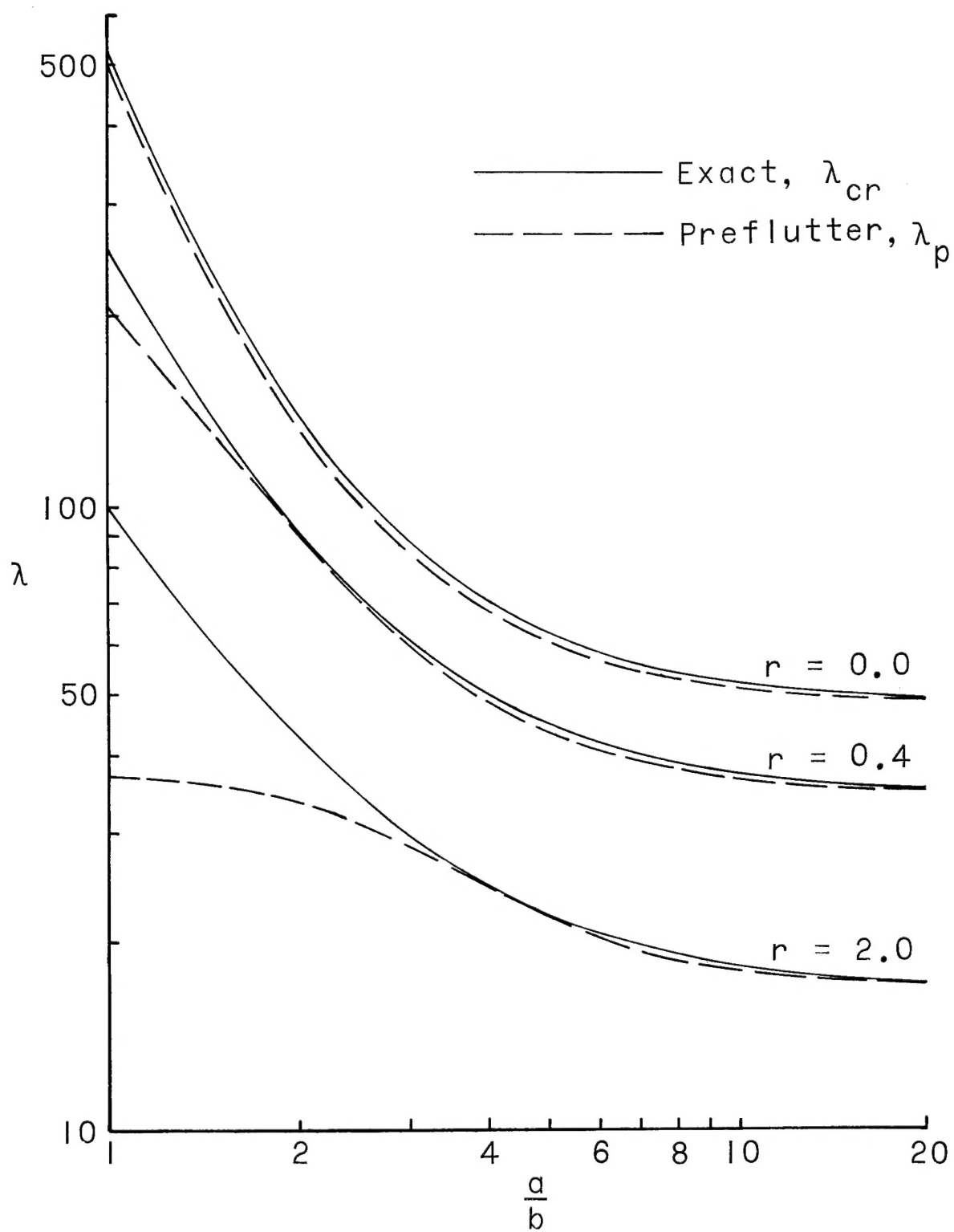


Figure 9.- Comparison of preflutter and exact flutter boundaries.  $k_x = 0$ .

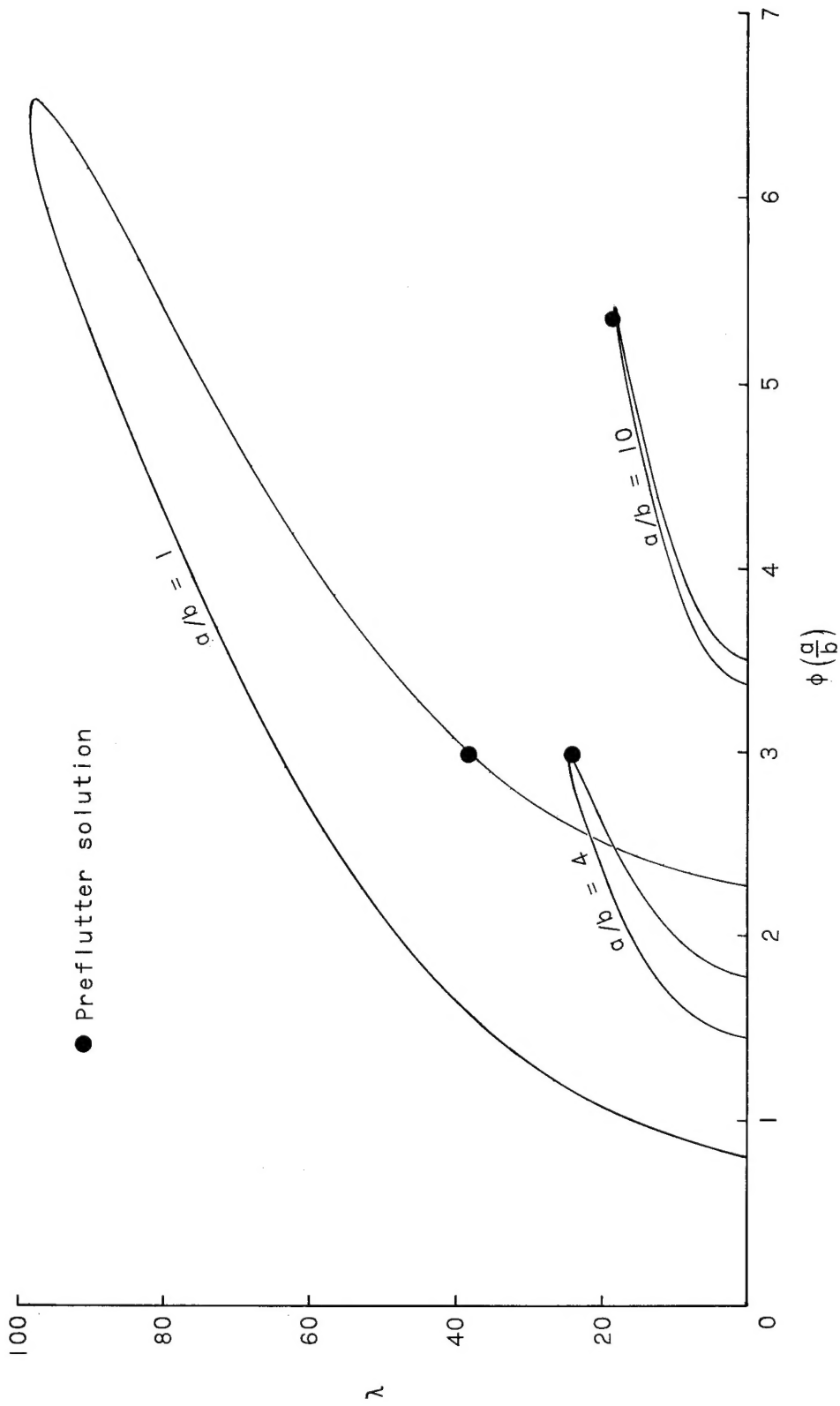
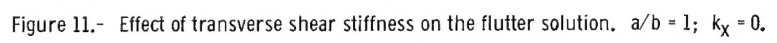


Figure 10.- Location of preflutter solution on frequency loops.  $k_y = 0$ ;  $r = 2$ .



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